SINO-PLATONIC PAPERS

Number 320 December, 2021

A Proposed Mesopotamian Origin for the Ancient Musical and Musico-Cosmological Systems of the West and China

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SINO-PLATONIC PAPERS

FOUNDED 1986

ISSN
2157-9679 (print) 2157-9687 (online)

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A Proposed Mesopotamian Origin for the Ancient Musical and Musico-Cosmological Systems of the West and China

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ABSTRACT

Cuneiform tablets translated since the early 1960s have allowed music archaeologists to reconstruct the Mesopotamian tonal system. The consensus is that, for over a millennium, from at least 1800 BC onward, the Mesopotamians used seven diatonic modes — scales that are closely related to the Western, seven-note major scale. Furthermore, the Mesopotamian method of re-tuning stringed instruments from one mode to another, created five extra pitches that correspond with what, in the West, are called sharps and flats. On one particular tablet, these notes are understood to be generated as consecutive "fifths," suggesting that the "cycle of fifths" — a circular arrangement of the twelve pitches that make up an octave — was known to the Mesopotamians.

Fundamental to the Mesopotamian re-tuning cycle is the sequence 4,1,5,2,6,3,7 — and its inversions — which, among other things, indicates the order of the strings to be re-tuned. As will be shown, this sequence is also embedded in the ancient Chinese *sanfen sunyi* (三分損益) method, a method that documents mathematical generation of the cycle of fifths and that was used, as early as the third century BC, to create the *shí'èr lǜ* (十二律) or "twelve-pitch" system. This twelve-pitch system was in turn used to create scales having fewer than twelve notes, most notably a five-note scale and a seven-note scale — the modes of which are identical to those used in Mesopotamia.

In both Mesopotamia and China, music was central to cosmology. In China, the concept of *ganying* (感應), or "correlative resonance," was the rationale behind a system of intuitive correspondences whereby associations were made between items of different categories. For example, each planet was linked to a specific musical note, direction, color, etc. As will be shown, the sequence 4,1,5,2,6,3,7, which forms the basis of the Mesopotamian tonal system, was used, in the West, to create a similar system of analogies, suggesting that the phenomenon of "correlative cosmology," generally thought to be native to China, may, in fact, have had a Near Eastern origin.

This paper will propose, first, that the theoretical musical system of ancient China was derived from the Mesopotamian tonal system during the first half of the first millennium BC, and, second, that the tradition of correlative thinking central to Chinese cosmology originated in the Near East and was transmitted to China during the second half of the first millennium BC.

Keywords: Music, Musicology, Stringed Instruments, Ratio, Proportion, Analogy, Logos, Archaeology, History, Mathematics, Cosmology, Philosophy, History of Religions, Shamanism, Magianism, Music of the Spheres, Correlative Cosmology, Ganying, Mesopotamia, Elam, Persia, Media, China, Greece, Origin of the Major Scale, Origin of the Seven-day Week, Pythagoras, Plato, Aristotle, Seven, Twelve

CONTENTS

PART ONE:

A COMPARISON OF THE MUSICAL SYSTEMS OF MESOPOTAMIA, THE WEST, AND CHINA

1. INTRODUCTION

In 1962, Joseph Needham and Kenneth Robinson put forward the hypothesis that the musical systems of ancient China and ancient Greece had a common origin: Mesopotamia.

> [The] hypothesis is that the Babylonians discovered the mathematical laws governing the … length of strings forming the octave, fifth, and fourth intervals. This knowledge spreading both west and east was used by the Greeks and the Chinese independently, the former for constructing their acoustic theory by subdivision first of the octave and later of the tetrachord, the latter for developing a spiral of notes by an alternating series of fifths and fourths from a given fundamental.[1]

Yet Needham and Robinson admitted that their hypothesis could not, at the time, be proven, for not much was then known about the musical system of Mesopotamia: "a Babylonian origin for these discoveries is hypothetical, for of Babylonian music we know very little."[2]

Since the early 1960s, however, much has been learned about the Mesopotamian tonal system. Consequently, many scholars now believe that "the early Greeks took the principles of their music from the Near East."[3] Furthermore, since the mid-1980s the discovery of ancient Caucasoid settlements in the Tarim Basin of East Central Asia[4] has indicated that contact between the Near East and this region may have been established as early as 1800 BC: among the artifacts excavated in the Tarim Basin are twenty-three harps[5] — some dating to as early as 1000 BC — that resemble the angular harps developed in Mesopotamia circa 1900 BC. In light of these discoveries, this paper will revisit the hypothesis that the

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1 Joseph Needham and Kenneth Robinson, *Science and Civilisation in China*, Vol. 4, *Sound* (*Acoustics*) (Cambridge: Cambridge University Press, 1962), 181.

2 Needham and Robinson, *Science and Civilisation*, 177.

3 O. R. Gurney, "Babylonian Music Again," *Iraq* 56 (1994): 101–106.

4 Victor. H. Mair, "The Mummies of East Central Asia," *Expedition* 52 (no. 3) (Winter, 2010): 23–32.

5 https://languagelog.ldc.upenn.edu/nll/?p=49511

musical system of ancient China was derived from the Mesopotamian tonal system, suggesting the route of transmission to have been via the Tarim Basin and the Hexi Corridor, into the East Asian Heartland, beginning in the early first millennium BC.

2. ANCIENT STRINGED INSTRUMENTS ACROSS ASIA

The first stringed instrument is thought to have been the musical bow (Figure 1), which may have evolved from the hunting bow circa 13,000 BC. The musical bow is believed by some archaeologists to be the ancestor of the first multiple-string instruments: the harp and the lyre.

Figure 1. Musical Bow

The harp is believed to have predated the lyre, as the lyre is essentially a harp with the addition of a bridge: a piece of wood that supports the strings and transmits their vibrations to the body of the instrument. Evidence of the first type of harp, the arched harp, appears simultaneously around 3000 BC in Mesopotamia (Iraq) and Elam (Iran). From the Near East, the arched harp then disseminated: to India (and from there, much later, to China); to Greece; and to Egypt (Figure 2).

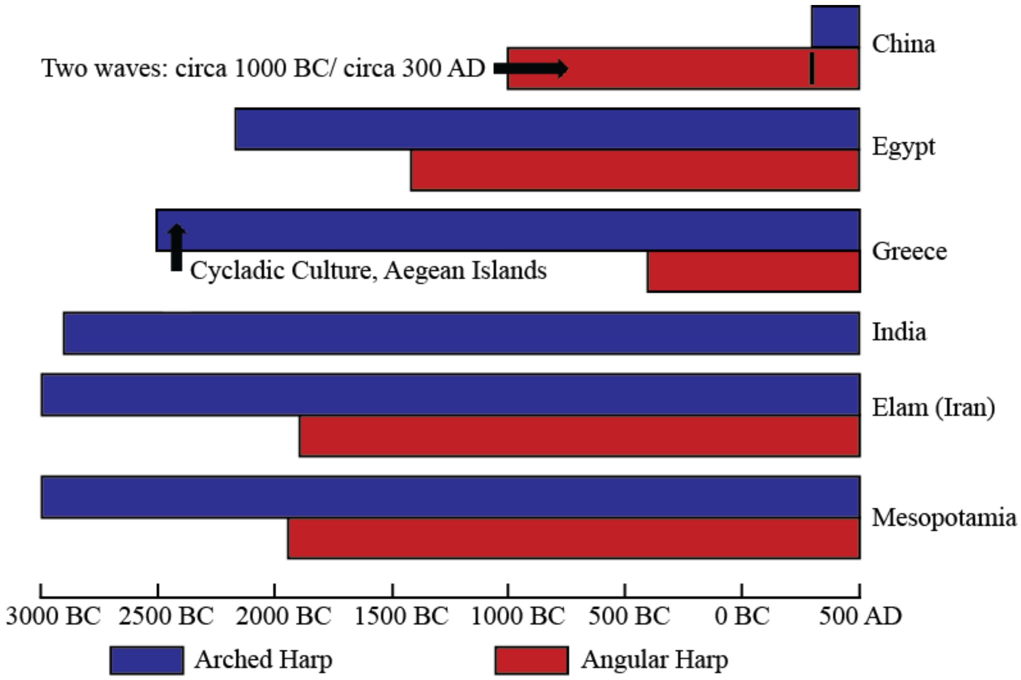

Figure 2. Dissemination of the Arched Harp and Angular Harp

Figure 3 shows a depiction of an arched harp having four strings, found at the city of Ur, in Lower Mesopotamia. Figure 4 shows a depiction of an arched harp also strung with four strings, found near the ancient Elamite city of Susa. The two sites are approximately 325 kilometers apart.

Figure 3. Detail from Cylinder Seal Depicting Arched Harp, Ur, Mesopotamia, 2800 BC (University of Pennsylvania Museum, Philadelphia)

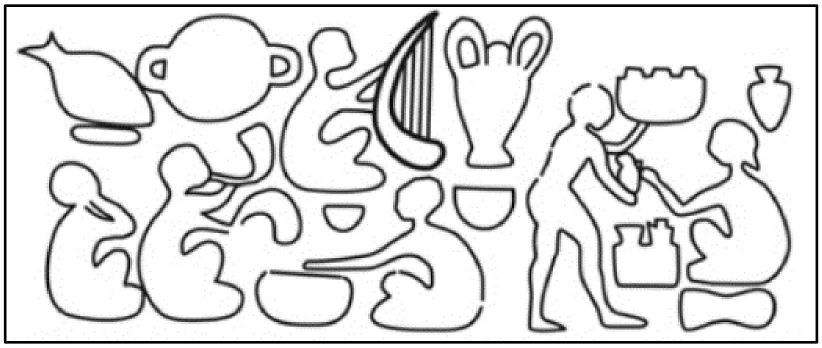

Figure 4. Detail from Cylinder Seal Depicting Arched Harp, Choghā Mīsh, Iran, 3100 BC (Louvre, Paris)

In both Mesopotamia and Elam, the arched harp was strung with a varying number of strings — often more than ten. An example is the reconstructed arched harp in Figure 5, found in the royal cemetery of Ur, which shows evidence of having been strung with thirteen strings.

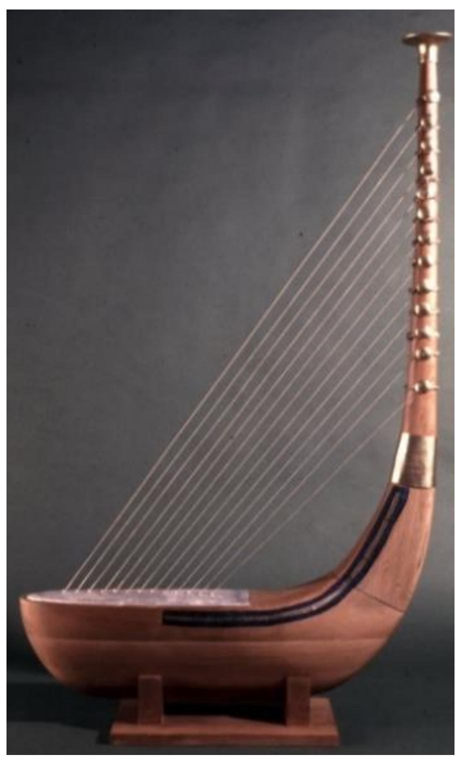

Figure 5. Reconstructed Arched Harp, Ur, Mesopotamia, 2600 BC (British Museum, London)

Over a thousand years after the first evidence of the arched harp, the angular harp appeared in Mesopotamia circa 1900 BC, spreading rapidly to Elam. The angular harp then reached Egypt circa 1400 BC, and Greece circa 450 BC. Generally, the arrival of the angular harp rendered the arched harp obsolete. In India, however, the arched harp was never replaced by the angular harp and remained in use, spreading to western China circa 300 AD. The angular harp also arrived at the western borders of China, but in two different waves (see Figure 2). The first wave (only recently discovered) occurred circa 1000 BC; the second wave began circa 300 AD.

While the body of the arched harp was usually a single piece of wood, the angular harp was made from two pieces of wood, attached at a right angle. There developed two types of angular harps: vertical and horizontal. A wall relief from 865 BC from Nimrud, a city in Upper Mesopotamia, shows two horizontal, angular harps, each strung with nine strings (Figure 6).

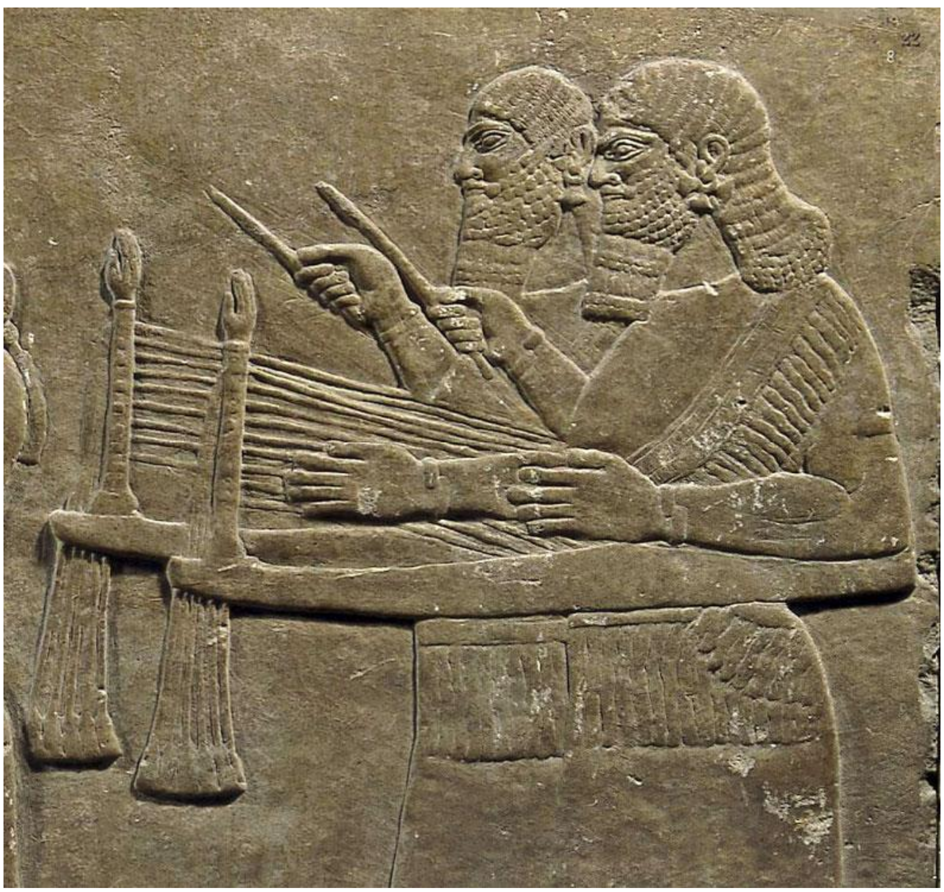

Figure 6. Wall Relief Showing Assyrian Angular Harps, Upper Mesopotamia, 865 BC (British Museum, London)

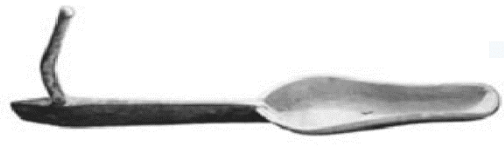

Figure 7. Angular Harp, Zhāgǔnlǔkè, 400–500 BC

Figure 8. Angular Harp, Yánghǎi, circa 1000 BC

Since 1996, twenty-three harps[6] having a similar design (two are shown in Figure 7 and Figure 8), dating from 1000 BC to 200 BC, have been found in the Tarim Basin, in Xinjiang Uyghur Autonomous Region, an area of western China (see map: Figure 9). The average length of these harps is between 60 cm and 70 cm — the approximate size of the harps depicted in Figure 6.

Before the first of the Xinjiang harps were excavated, three similar harps had already been found: two in Pazyryk, in the Altai Mountains, dated circa 350 BC, and one in Olbia, on the coast of the Black Sea, dated circa 100 AD.[7] It is probable, therefore, that the angular harp was disseminated widely across Eurasia.

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6 He Zhiling and Wang Yongqiang 贺志凌 王永强, "The Musical Archaeological Study of *Konghou* in Hami Wupu Eschernan Cemetery," *Chinese Music* (2018): 117–122. (English translation forthcoming in *Sino-Platonic Papers.*)

7 Bo Lawergren, "Angular Harps through the Ages: A Causal History," in *Studien zur Musikarchäologie VI*, ed. Arnd Adje Both, Ricardo Eichmann, Ellen Hickmann, and Lars-Christian Koch, *Orient-Archäologie 22* (Rahden, 2008): 276.

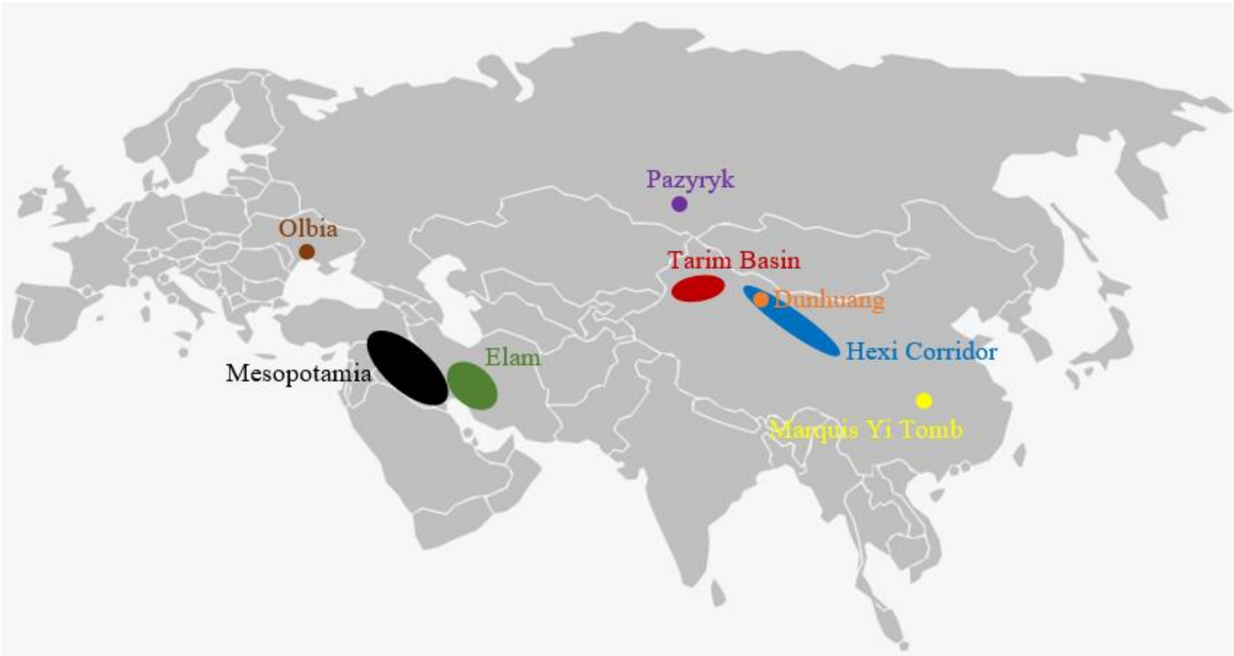

Figure 9. Map of Eurasia, Showing Various Locations Related to the Dissemination of the Angular Harp

In its first wave of dissemination, the angular harp reached Xinjiang over a millennium before the arched harp, which arrived from India, with Buddhism, circa 300 AD. Consequently, the specific design of the angular harp excavated in the Tarim Basin is not represented in any of the Buddhist cave paintings of the Silk Road era that show various depictions of harps.

Nevertheless, Silk Road era paintings (for example, Figure 10) do show some angular harps having seven strings — a number that, as we will see, was connected with music in Mesopotamia. The painting in Figure 10 (386–535 AD) is in the Dunhuang Caves, located near the northwestern end of the Hexi Corridor — a narrow stretch of arable plain, one thousand kilometers in length, that provides the easiest route from Xinjiang into China (see map: Figure 9). The Dunhuang Caves are only five hundred kilometers from Hami, the closest Xinjiang site at which angular harps have been found.

Figure 10. Seven-stringed Angular Harp, Mogao Grotto #431, Dunhuang, Gansu Province, China

To summarize, there were two types of harps that reached western China — the arched harp and the angular harp. Yet, for both these types of harp there was but a single Sinitic term: *kōnghóu* (空侯/箜篌). Several scholars believe that this word is related to the Sogdian words for harp: *cngry'* /*čangaryā*/ and *cyngry'* /*čingaryā*/ — as well as the Persian: چنگ (*čang*). Thus, according to V. Mair, "we have both the actual instrument and the borrowed Sinitic transcription of an apparently Iranian name for it."[8]

There is no archaeological proof that the specific type of angular harp excavated in Xinjiang made its way deeper into China. Nevertheless, music archaeologist B. Lawergren suggests that the Xinjiang harps had an influence on the design of later Chinese stringed instruments. Specifically, Lawergren proposes that the Xinjiang harps "inspired the shape of the *qin*" and that this "influence played out over a relatively small space and brief time: in Xinjiang during the first half of the first millennium BCE."[9]

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8 https://languagelog.ldc.upenn.edu/nll/?p=49511

9 Bo Lawergren, "Western Influences on the Early Chinese Qin-Zither," *Bulletin of the Museum of Far Eastern Antiquities* 75 (2003): 91.

Figure 11. *Qin* and Tuning Pegs, Tomb of Marquis Yi of Zeng, 433 BC (Hubei Provincial Museum, China)

The *qin* is the most revered of Chinese instruments. A handful of *qin* have been unearthed in archaeological sites dating from the fifth to second centuries BC. The oldest of these (Figure 11) is from the tomb of Marquis Yi (see map: Figure 9) and shows evidence of having had ten strings. In suggesting a relationship between the angular harps excavated in the Tarim Basin and the ancient Chinese *qin*, Lawergren notes a similarity in the shape of the two instruments:

> [Harps] in the Xinjiang region were asymmetrical. Half their bodies were slim like necks, the other half looked like a body with sloping shoulders. This structure is reminiscent of the ancient *qin*. Moreover, the playing position is horizontal for both instruments. Most likely, the harp inspired the shape of the *qin*.[10]

Lawergren also notes that, although later *qin* are an average length of approximately 120 cm, the earliest *qin* are much shorter — the Marquis Yi *qin* is 67 cm — and therefore similar in length to the Tarim Basin angular harps.

The tuning pegs of Marquis Yi's *qin* were wooden tubes (see Figure 11), each with an end that fit a "key" with a square socket (resembling a modern screwdriver) — required because the pegs were too close together to be turned with the fingers. Over time, the need for a key became obsolete because as the *qin* attained its full length the pegs became, proportionately, farther apart.

The key used with the Marquis Yi's *qin* was not found. Nevertheless, several ancient keys have

10 Lawergren, "Influences on the Qin-Zither," 91.

survived because, unlike the wooden instruments that they were used with, the keys were made of bronze. Figure 12 shows a drawing by Lawergren, detailing several ancient *qin* bronze keys, dating from the fifth century BC to the second century BC. Lawergren notes that "many of the motifs on tuning keys are closely associated with the art of the Eurasian steppes in north and northwest China," but that "the human-headed goat (see fig. 3.9.3e), a common motif at Persepolis in ancient Iran, suggests more distant connections."[11]

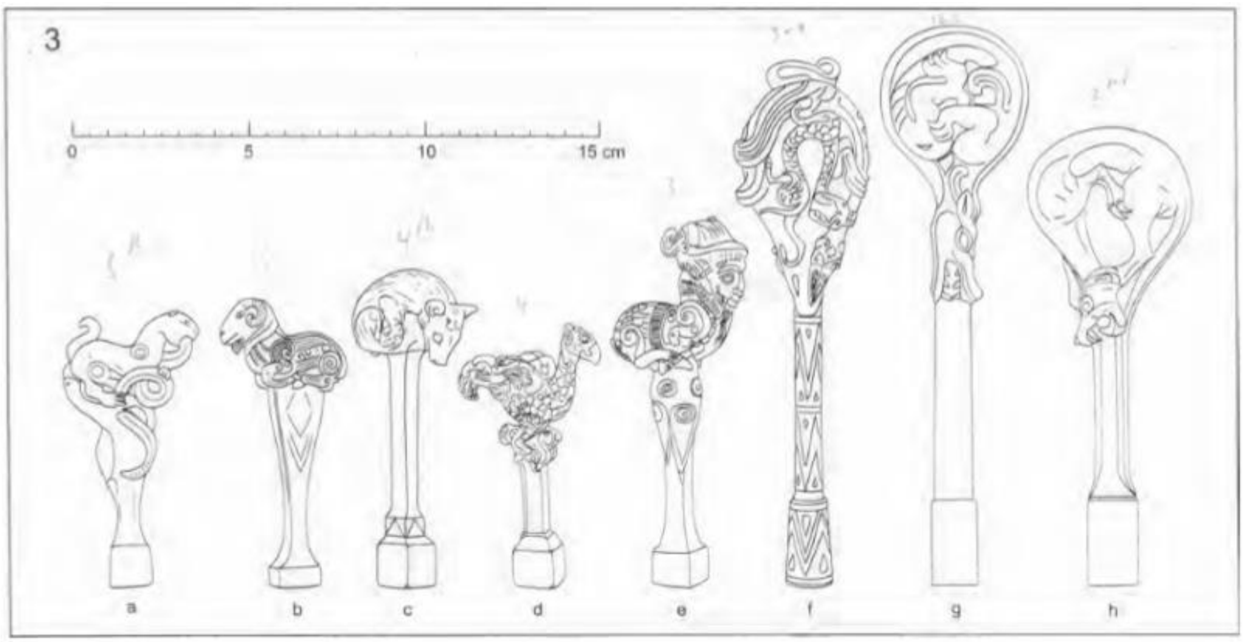

Figure 12. Ancient *Qin* Bronze Tuning Keys (Figure 3.9.3 from B. Lawergren, "Strings." In Jenny F. So, *Music in the Age of Confucius*, 78

Lawergren also looks at textual evidence, noting that the *Chu Ci*, a southern text of the third century BC, mentions the *se* — an ancient Chinese zither (the *qin* is classified as a "half-tube zither") — six times, but makes no mention of the *qin*. On the other hand, northern texts, he finds, mention the *se* and *qin* more or less equally.

Finally, Lawergren notes that both the Xinjiang angular harps and the ancient Chinese *qin*

11 Bo Lawergren, "Strings," in *Music in the Age of Confucius*, ed. Jenny F. So, 77 (Washington DC: Freer Gallery of Art and Arthur M. Sackler Gallery / Seattle and London: University of Washington Press, 2000).

appear to have had a relatively small number of strings — as compared to the *se*, for example, which usually had twenty-five strings. For although the Marquis Yi *qin* shows evidence of having had ten strings, the number of strings usually associated with the instrument is seven. Similarly, the number of strings on the Tarim Basin harps "seem[s] to have been low, such as 3, 5, 5, and 15."[12] Generally, Lawergren believes that the Xinjiang harps were strung with five strings.[13]

Figure 13. UET VII 74, 1800 BC (Iraq Museum, Baghdad)

Lawergren interprets the fact that some of the Tarim Basin harps show evidence of having been strung with five strings to indicate that knowledge of the tuning system that had been used in conjunction with the instrument in Mesopotamia — a system of seven-note, diatonic[14] scales — had been lost by the time the instrument reached Xinjiang:

12 Lawergren, "Influences on the Qin-Zither," 95.

13 Lawergren, "Angular Harps," 264.

14 A diatonic scale is a scale containing seven notes that are separated by the intervals tone, tone, semitone, tone, tone, tone, semitone (or inversions of this pattern). An example is the scale of C major: C-(tone)-D-(tone)-E-(semitone)-F-(tone)-G-(tone)-A-(tone)-B-(semitone)-C.

> The Mesopotamian nine-stringed harp was famously used to demonstrate Mesopotamian tuning theory, which used diatonic scales like ours. But having moved east, it ended up with five strings, and this probably implies it had severed any association with the Mesopotamian tuning theory. It arrived in Xinjiang without the theoretical framework. [15]

The "famous" demonstration of Mesopotamian tuning theory that Lawergren refers to was the discovery, in 1968,[16] of the "tuning text" (tablet UET VII 74, Figure 13). This tablet gives instructions on how to tune the nine-stringed *sammû* to seven modes, or tunings — each having seven notes — and played an important role in the emerging field of music archaeology. Lawergren identifies the *sammû* as the angular harp, but, in fact, the *sammû* is now understood to be a lyre:

> For a long time, scholars were not clear whether the *sammû* was a lyre or a harp.... But over the last 30 years, opinion has hardened in favour of its being a lyre, and this is the interpretation firmly adopted in the great *Chicago Assyrian Dictionary*. At the time when Sumero-Babylonian music theory was formulated, the word would presumably have denoted the large Sumerian type of lyre with its soundbox embellished at one end with the head of a bull or cow....[17]

The first lyres appeared in Mesopotamia around 2500 BC — after the arched harp was in use, but before the invention of the angular harp. Like the harps described earlier, lyres had a varying number of strings: "Early Mesopotamian representations of lyres show them with string numbers ranging from three to twelve."[18] In Elam, at the same period, bull-headed lyres are also documented.

15 Lawergren, "Angular Harps," 265.

16 O. R. Gurney, "An Old Babylonian Treatise on the Tuning of the Harp," *Iraq* 30 (1968): 229–233.

17 M. L. West, "The Babylonian Musical Notation and the Hurrian Melodic Texts," *Music and Letters* 75 (issue 2, 1994): 166.

18 https://www.penn.museum/sites/expedition/the-musical-instruments-from-ur-and-ancient-mesopotamian-music/

Figure 14 shows a Mesopotamian lyre strung with eleven strings. Figure 15 shows an Elamite depiction of a bull-headed lyre strung with six(?) strings.

Figure 14. Queen's Lyre, Ur, 2500 BC (British Museum, London)

As mentioned earlier, cuneiform tablets that describe the Mesopotamian tonal system have been deciphered. However, to date, no textual evidence describing a different Elamite system has been found. Because the musical instruments of the two cultures were so similar, it is the general consensus that the tonal system used in Mesopotamia — which is documented by tablets as being in use throughout Mesopotamia for over a thousand years — was also used in neighboring Elam.

Let's summarize. In both Mesopotamia and Elam, harps and lyres were strung with a varying number of strings. A similar situation existed in the East: in both Xinjiang and China the number of strings on harps and *qin* varied greatly. We see, therefore, that the assumption that because some of the Xinjiang harps show evidence of having been strung with five strings their connection with Mesopotamian tuning theory was lost is unfounded. For in the Near East itself, harps and lyres are depicted as having a varying number of strings — sometimes even fewer than five.

It is the consensus among music archaeologists that the Mesopotamian tonal system used diatonic scales — seven-note scales that are the ancestor of the modern, Western major scale. (We will look at the evidence for this shortly.) But the question arises: how could instruments having fewer than seven strings have been related to this seven-note system?

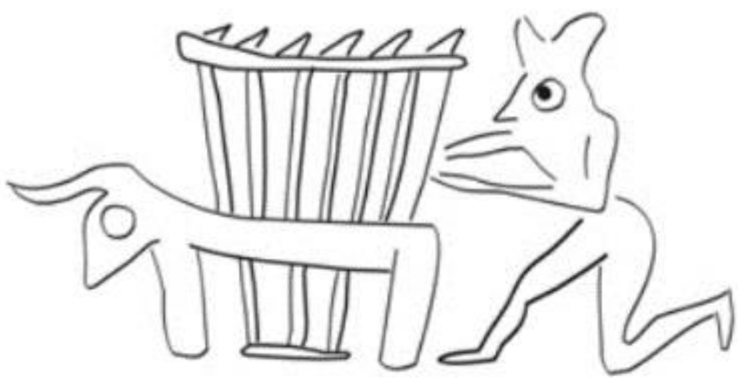

Figure 15. Cylinder Seal from Susa, 2600 BC (Tehran Museum, Iran)

Although the tuning instructions described on UET VII 74 do, indeed, generate seven-note diatonic scales, *they do so by selecting seven notes from a cycle of twelve notes*. This has led scholars to deduce "that the cycle of 5ths was known"[19] to the Mesopotamians. It is important, therefore, that, from the third century BC, the Chinese also used twelve pitches, generated by the cycle of fifths. This *shí'èr lǜ* or "twelve-pitch" system was used to create scales having fewer than twelve notes, most notably a pentatonic scale and a heptatonic scale. As we will see, the modes of this ancient Chinese heptatonic scale are identical with the modes used by the Mesopotamians. Furthermore, we will see that the

19 https://www.penn.museum/sites/expedition/the-musical-instruments-from-ur-and-ancient-mesopotamian-music/

mathematics by which the ancient Chinese derived the *shí'èr lǜ* pitches has a direct relationship to the Mesopotamian tonal system.

3. THE ANCIENT CHINESE TONAL SYSTEM: THE *SHÍ'ÈR LǛ*

The *Lǚshì chūnqiū* (春秋) is an ancient Chinese encyclopedic work that dates from 239 BC. In Chapter Five, *Gu Yue* (古樂), or "Music of the Ancients," the following story appears:

> In the past, the Yellow Sovereign commanded Ling Lun to create pitch-standards. Ling Lun, having passed through the western regions of Dàxià, then went to the shady northern slopes of the Kunlun Mountains. He selected bamboo from the valley of Xiexi which had hollows and walls of uniform thickness. Cutting it between two nodes to a length of 3.9 inches, he blew on it and fixed its sound as the note *gōng* for the Yellow Bell [*Huáng Zhōng*] pitch-standard.... He then made the twelve bamboo tubes, one after the other.... These he harmonized with the fundamental note *gōng* of Yellow Bell. The note *gōng* of Yellow Bell can be used to generate all the other notes....[20]
>
> 昔黃帝令伶倫作為律, 伶倫自大夏之西, 乃之阮腧之陰，取竹於懈谿之谷， 以生空竅厚鈞者，斷兩節間，其長三寸九分, 而吹之，以為黃鐘之宮，吹曰[舍少〕，次制十二筒，以之阮隃之下，聽鳳凰之鳴, 以別十二律，其雄鳴為六，雌鸣亦六 以比黃鐘之宮適合, 故曰[黃鐘之宮，律呂之本]

This story recounts the mythical journey taken by Ling Lun to establish the *shí'èr lǜ* — the twelve pitches of the ancient Chinese tonal system. "Dàxià," the region that Ling Lun passed through, is the name given in antiquity by the Han Chinese to Tukhara or Tokhara: the main part of Bactria, in what is now northern Afghanistan and parts of southern Tajikistan and Uzbekistan. The name "Kunlun," which dates back at least to the first millennium BC, was used to denote a semi-mythical mountain range and was most often associated with the chain that runs along the northern edge of the Tibetan Plateau, south of the Tarim Basin.[21]

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20 *Lǚshì chūnqiū* (春秋), Chapter Gu Yue (古樂), in *The Annals of Lü Buwei: A Complete Translation and Study*, trans. J. Knoblock and J. Riegel (Stanford University Press, 2000), 147.

21 https://languagelog.ldc.upenn.edu/nll/?p=50380&utm_source=rss&utm_medium=rss&utm_campaign=kunlun-the-

The *Lǚshì chūnqiū* also describes, in Chapter Six, "Tonal Pitch" (音律), the mathematical procedure that generates this twelve-pitch system — the *sanfen sunyi* (三分損益) method:

The *Huáng Zhōng* pitch generates the *Lín Zhōng* pitch.
The *Lín Zhōng* pitch generates the *Tài Cù* pitch.
The *Tài Cù* pitch generates the *Nán Lǚ* pitch.
The *Nán Lǚ* pitch generates the *Gū Xiǎn* pitch.
The *Gū Xiǎn* pitch generates the *Yìng Zhōng* pitch.
The *Yìng Zhōng* pitch generates the *Ruí Bīn* pitch.
The *Ruí Bīn* pitch generates the *Dà Lǚ* pitch.
The *Dà Lǚ* pitch generates the *Yí Zé* pitch.
The *Yí Zé* pitch generates the *Jiá Zhōng* pitch.
The *Jiá Zhōng* pitch generates the *Wú Yì* pitch.
The *Wú Yì* pitch generates the *Zhòng Lǚ* pitch.

The addition of 1/3 of its value gives rise to the up-generation. The subtraction of 1/3 of its value gives rise to the down-generation. *Huáng Zhōng*, *Dà Lǚ*, *Tài Cù*, *Jiá Zhōng*, *Gū Xiǎn*, *Zhòng Lǚ*, and *Ruí Bīn* are obtained by up-generation. *Lín Zhōng*, *Yí Zé*, *Nán Lǚ*, *Wú Yì*, and *Yìng Zhōng* are obtained by down-generation.[22]

黃鐘生林鐘，林鐘生太蔟，太蔟生南呂，南呂生姑洗, 姑洗生應鐘，應鐘生蕤賓，蕤賓生大呂，大呂生夷則, 夷則生夾鐘，夾鐘生無射，無射生仲呂
三分所生益之一分以上生, 三分所生去其一分以下生

origins-and-meanings-of-a-mysterious-place-name

22 *Lǚshì chūnqiū* (春秋), Chapter Six, "Tonal Pitch" (音律), in *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts*, trans. C. Y. Chen (Hong Kong University Press, 1996), 52.

黄鐘，大呂，太蔟，夾鐘，姑洗，仲呂，蕤賓，為上
林鐘，夷則，南呂，無射，應鐘，為下

We'll look at the *sanfen sunyi* method shortly, but first let's list the *shí'èr lǜ* in the order that they are first given in the text: *Huáng Zhōng* ("Yellow Bell"), *Lín Zhōng* ("Forest Bell"), *Tài Cù* ("Great Budding"), *Nán Lǚ* ("Southern Regulator"), *Gū Xiǎn* ("Maid Purity"), *Yìng Zhōng* ("Resonating Bell"), *Ruí Bīn* ("Luxuriant"), *Dà Lǚ* ("Great Regulator"), *Yí Zé* ("Equalizing Rule"), *Jiá Zhōng* ("Compressed Bell"), *Wú Yì* ("Tireless"), and *Zhòng Lǚ* ("Mean Regulator").[23]

The above order is reproduced in column 1 of Table 1, where the pitches that are up-generated (as described later in the text) are printed in red, and those that are down-generated are printed in green. In column 2, the up-generated pitches are numbered consecutively, as are the down-generated pitches in column 3. In column 1 of Table 2, the twelve pitches are listed in the second order that they are given in the text. Column 2 of Table 2 lists the number of each up-generated pitch, as given in Table 1. Column 3 lists the number of each down-generated pitch, as given in Table 1. Notice the sequence of numbers in column 2 of Table 2: 1,5,2,6,3,7,4. As we'll now see, this sequence and its inversions form the basis of the Mesopotamian tonal system.

Table 1. *Shí'èr lǜ* Order in Part 1 of *Lǚshì chūnqiū* text

***Shí'èr lǜ* Order: Part 1 of text**	**Up-Generations in order**	**Down-Generations in order**
Huáng Zhōng	1	
Lín Zhōng		1
Tài Cù	2	
Nán Lǚ		2
Gū Xiǎn	3	
Yìng Zhōng		3
Ruí Bīn	4	
Dà Lǚ	5	

23 English translation of pitch names from Knoblock and Riegel, *The Annals of Lü Buwei*, 157.

***Shí'èr lǜ* Order: Part 1 of text**	**Up-Generations in order**	**Down-Generations in order**
Yí Zé		4
Jiá Zhōng	6	
Wú Yì		5
Zhòng Lǚ	7	

Table 2. *Shí'èr lǜ* Order in Part 2 of *Lǚshì chūnqiū* text

***Shí'èr lǜ* Order: Part 2 of text**	**Up-Generations in order**	**Down-Generations in order**
Huáng Zhōng	1	
Dà Lǚ	5	
Tài Cù	2	
Jiá Zhōng	6	
Gū Xiǎn	3	
Zhòng Lǚ	7	
Ruí Bīn	4	
Lín Zhōng		1
Yí Zé		4
Nán Lǚ		2
Wú Yì		5
Yìng Zhōng		3

4. THE MESOPOTAMIAN TONAL SYSTEM: CUNEIFORM TABLET CBS 1766

Since the early 1960s a handful of cuneiform tablets have thrown light on the nature of the Mesopotamian tonal system. Written on two of these tablets — UET VII 74 (Figure 13) and CBS 1766 (Figure 16) — are inversions of the sequence 1,5,2,6,3,7,4: the sequence noted in Table 2.

We will look at the tuning instructions on UET VII 74 shortly, but first let's examine tablet CBS 1766, the date of which is uncertain but is thought to be between 1780 BC and 600 BC. At the top left corner of CBS 1766 is drawn a heptagram, inscribed in two concentric circles. At the points of the heptagram are written, in cuneiform script, the numbers 1 to 7, in a clockwise direction. Also written at the points of the heptagram are the names of seven of the strings of the *sammû*, the Mesopotamian lyre. Below the diagram is a table with four legible columns and four partially legible columns (Figure 16). In the third column is written the sequence 1,5,2,6,3,7,4 — the sequence noted in Table 2. In the other legible columns are written inversions of this sequence.

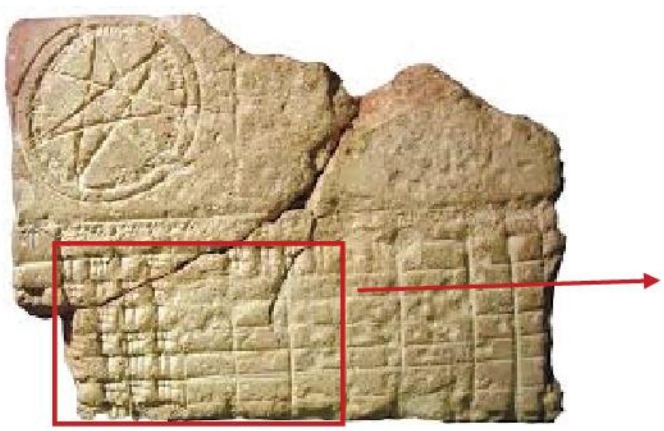

Figure 16. CBS 1766, 1780–600 BC (University of Pennsylvania Museum)

2	6	1	7	5	4	7	2
6	3	5	4				
3	7	2	1	not legible			
7	4	6	5				
4	1	3	2				
1	5	7	6				
5	2	4	3				

Table 3. Table from CBS 1766

The diagram and the table are related: following the diagonals of the heptagram and listing the number at each point generates the sequence 1,5,2,6,3,7,4, and its inversions (Figure 17).

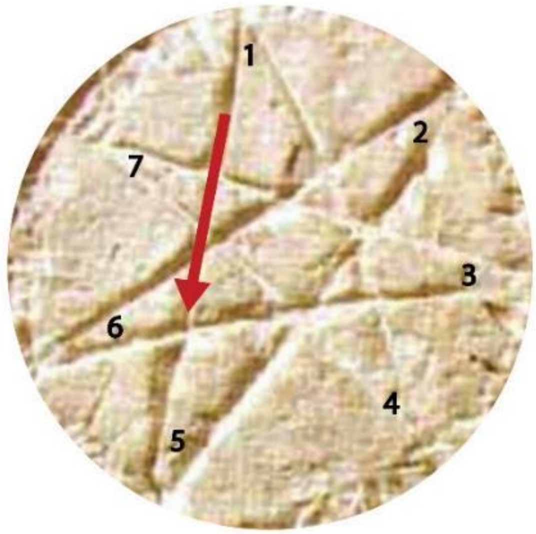

Figure 17. The Heptagram on CBS 1766, detail. Starting at the red arrow and following the diagonals of the heptagram generates the sequence 1,5,2,6,3,7,4: the sequence given in column 2 of Table 2 and column 3 of Table 3

What we will see now is that the sequence 1,5,2,6,3,7,4 can be derived mathematically from the ancient Chinese *sanfen sunyi* method and, also, that the diagram on CBS 1766 can be shown to be a geometric representation of this method.

5. THE ANCIENT CHINESE TONAL SYSTEM: THE *SANFEN SUNYI* METHOD

As we saw earlier, according to Ling Lun's story from the *Lǚshì chūnqiū*, the *shí'èr lǜ* were made from bamboo reeds. As modern acoustics has revealed, however, the resonating air column within a reed or pipe is slightly longer than the pipe itself, causing the relationship between wavelength and frequency (pitch) to be obscured, thereby making it difficult to use simple mathematical ratios to generate harmonically related pitches from pipes. With regards to a string, however, string length (wavelength) and pitch (frequency) are exact reciprocals, making it relatively easy to conceptualize the relationship between mathematical ratios and pitch. This fact suggests that the *shí'èr lǜ* pitches were most likely, originally, derived using a stretched string.

Evidence that harmonic ratios were originally explored using a stretched string is given by Wei Zhao (韋昭) in the third century AD. Commenting on a passage in the *Guo Yu* (fifth century BC) that makes reference to an instrument that resembles a monochord — a one-stringed instrument used to explore harmonic ratios — Wei Zhao states that this instrument was used to tune bells:

> "The term *jūn* (均) referred to here is ... an instrument which is seven *chǐ* 尺 (feet) in length mounted with string (or strings) and is used for tuning bells."[24]
>
> 均者，均鐘木，長七尺，有絃之，以均鐘者.

With this in mind, we will now apply the *sanfen sunyi* method (literally "a method of subtracting and adding thirds" — also referred to as the ancient Chinese up-and-down principle) to a stretched string, which we'll imagine to be strung on the *jūn*: the device described by Wei Zhao.

As we saw earlier, the first part of the *Lǚshì chūnqiū* text lists the twelve *shí'èr lǜ* pitches in this order: *Huáng Zhōng*, *Lín Zhōng*, *Tài Cù*, *Nán Lǚ*, *Gū Xiǎn*, *Yìng Zhōng*, *Ruí Bīn*, *Dà Lǚ*, *Yí Zé*, *Jiá Zhōng*, *Wú Yì*, *Zhòng Lǚ*, stating that each pitch is derived from the previous one.

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24 Guoyu (國語), Zhouyu (周語下), Section II, trans. C. Y. Chen, *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts* (Hong Kong University Press, 1996), 23.

The second part of the *Lǚshì chūnqiū* text then lists seven of the twelve pitches — those that are derived by "up-generation" (i.e., by adding 1/3 to the value of the previous pitch) — in a different order: *Huáng Zhōng*, *Dà Lǚ*, *Tài Cù*, *Jiá Zhōng*, *Gū Xiǎn*, *Zhòng Lǚ*, *Ruí Bīn*. Then the five pitches that are derived by "down-generation" (i.e., by subtracting 1/3 from the value of the previous pitch) are also listed — but also in a different order: *Lín Zhōng*, *Yí Zé*, *Nán Lǚ*, *Wú Yì*, *Yìng Zhōng*. We will now derive these pitches mathematically, using the *sanfen sunyi* method, and in so doing understand the origin of these alternate orders, given in the second part of the *Lǚshì chūnqiū* text.

In Western musical tradition, we call the note sounded by the whole string the *fundamental*. In the *Lǚshì chūnqiū*, the fundamental — the note from which all others are derived — is called *Huáng Zhōng*. We will assume the *Huáng Zhōng* to have a string length of 1 and we will assign to it, using Western musical notation, the name C (this is arbitrary: any pitch name could be given).

By dividing the whole string in half and plucking either half, we hear the octave of the fundamental (Figure 18) — in this case another C. Theoretically, this process can be continued indefinitely: repeatedly dividing the string in half generates increasingly higher octaves of the fundamental. But melodies cannot be made from multiple octaves of the *same* note. To create melodies, musicians must agree on notes to use *between* the octaves. In other words, they must create a scale. The instructions in the *Lǚshì chūnqiū* generate a scale using simple mathematics.

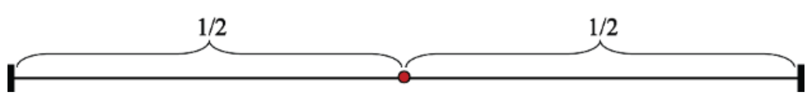

Figure 18. The Octave

The next simplest way to divide a string is in three parts. When 1/3 of the string is plucked, we hear a new note. When 2/3 of the string is plucked, we hear this same note, an octave lower. Why? Because 2/3 is twice the length of 1/3. In the West, we call the note sounded by 2/3 of the whole string the "fifth" (Figure 19). This nomenclature has to do with the structure of the major scale, for if the fundamental is assumed to be the first note of a major scale, plucking 2/3 of the whole string will sound the fifth note of that same scale. For example, G, the fifth note in the scale of C Major, is heard by plucking 2/3 of a string, the entire length of which sounds the note C.

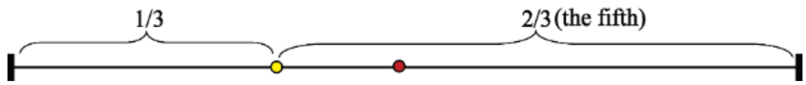

Figure 19. The Fifth

The first part of the *Lǚshì chūnqiū* instructions describes that a second pitch is generated from the *Huáng Zhōng*. This is done by "down-generation" — i.e., by subtraction of 1/3 of its value (1-1/3 = 2/3). The resulting pitch has 2/3 of the string length of the *Huáng Zhōng* and is called *Lín Zhōng*. It follows, therefore, that if *Huáng Zhōng* is C, then *Lín Zhōng* is the fifth of C: the note G.

To calculate the value of the third pitch, *Tài Cù*, we take the value of *Lín Zhōng* (2/3) and apply an "up-generation" (i.e., we add 1/3 of the value of *Lín Zhōng* to itself) to obtain 8/9 [because (2/3) + (1/3) × (2/3) = 2/3 + 2/9 = 6/9 + 2/9 = 8/9]. To understand the relationship between these two pitches, *Tài Cù* and *Lín Zhōng*, let's return to our discussion of the octave.

As described previously, dividing a string repeatedly in half generates a series of octaves. A series of fifths is generated by a similar process, but instead of repeatedly shortening the string by 1/2, it is shortened each time by 1/3. Although the two operations are similar, the results are different: where shortening by 1/2 generates ascending octaves of the *same* note, shortening by 1/3 generates a series of *different* notes, *each one the fifth of the note before it*. For example, D, the fifth of G, is heard by shortening the string length of G by 1/3 and plucking the remaining 2/3 (Figure 20). The resulting length is 2/3 – (2/3)1/3 = 4/9.

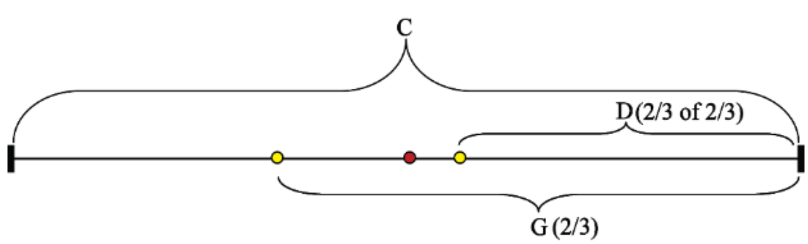

Figure 20. The Fifth of the Fifth

The fraction 4/9 equals .4444, which is less than 1/2. This means that D, with a string length of 4/9, lies outside the first octave. This is illustrated in Figure 21 by the yellow dot "D (4/9)" that can be seen to lie outside the blue rectangle that represents the first octave.

As mentioned earlier, the purpose of the instructions in the *Lǚshì chūnqiū* is to create a scale by filling the octave with notes. Yet already, in generating the fifth D, we have extended outside the first octave. The solution is to apply the octave rule, already discussed. For if *halving* the string length of a note sounds the same note in the next *higher* octave, *doubling* its string length must sound that note in the next *lower* octave. Therefore, to find the note D in the first octave, we simply multiply the string length of D (4/9) by 2 — or double it — to get the note D (8/9), as shown in Figure 21. This note — D, the fifth of G, with string length doubled and therefore having a value of 8/9 — is named, in the *Lǚshì chūnqiū* text, *Tài Cù*.

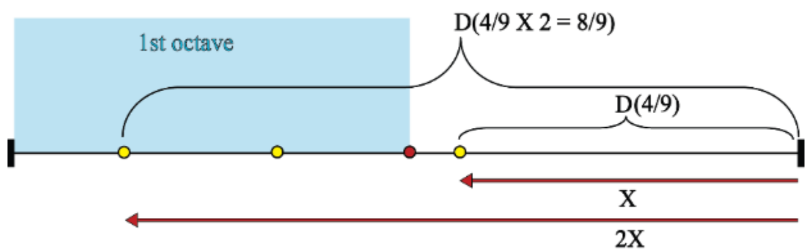

Figure 21. Moving D into the 1st Octave

	Chinese	Greek (Pythagorean)		Chinese	Greek (Pythagorean)
C	1	1	G	2/3	2/3
C♯	2048/2187	—	G♯	4096/6561	—
D	8/9	8/9	A	16/27	16/27
D♯	16384/19683	—	A♯	32768/59049	—
E	64/81	64/81	B	128/243	128/243
F	131072/177147	—	C	262144/531441	—
	—	3/4		—	1/2
F♯	512/729	—			

Figure 22. A Comparison of the Proportions of the Chinese and Greek (Pythagorean) Scales (Table 49 from Needham and Robinson, *Science and Civilisation in China*, 4: 175)

If we were to continue with the instructions in the *Lǚshì chūnqiū* we would generate the values given in Figure 22, which is a table reproduced from Needham and Robinson’s *Science and Civilisation in China*. Notice that Needham and Robinson correlate the *Huáng Zhōng*, *Lín Zhōng*, and *Tài Cù* pitches with the notes C, G, and D, respectively, as we have done. Their values for these notes also correspond with ours: C = 1; G = 2/3; D = 8/9. Needham and Robinson also give values for the pitches in the ancient Greek (Pythagorean) diatonic scale, which we will refer to later. [25]

Let’s look again at the formulas given in the *Lǚshì chūnqiū* for up-generation and down-generation. To make an up-generation 1/3 is added to the value of the previous pitch; to make a down-generation 1/3 is subtracted from the value of the previous pitch.

25 Notice that Needham and Robinson give a value for a thirteenth note: C, the octave of the fundamental. They give the string length of this note, as calculated by the Chinese, as 262144/531441, but give its value, as calculated by the Greeks, as 1/2. Similarly, Needham and Robinson give the Chinese value for the note F as 131072/177147 but the value for F, as calculated by the Greeks, as 3/4. We’ll discuss these differences shortly.

At first glance, it appears that only two operations are used: addition and subtraction — but this is *not* the case. Why? Because to add (or subtract) 1/3, the value of 1/3 must first be calculated. Although this may be a simple mental exercise for relatively small numbers, the numbers generated after following a few steps of the *sanfen sunyi* method become quite large — as we see from Needham and Robinson's calculations in Figure 22. Therefore, in cases involving large numbers, calculating 1/3 of a specific value *must be done using division*. And since division must be used, we can also hypothesize that, in applying the *sanfen sunyi* method, ancient Chinese music theoreticians also made use of multiplication, which is simply the inverse of division.

For example, we saw earlier that the value given by the *sanfen sunyi* method for the pitch *Tài Cù* is 8/9. We arrived at this value by adding 1/3 of the value of the pitch *Lín Zhōng* to itself: [(2/3) + (1/3) × (2/3) = 2/3 + 2/9 = 6/9 + 2/9 = 8/9]. But this calculation is greatly simplified if we understand that the formula for an up-generation [(Pitch or P) + P(1/3)] is the equivalent of multiplying the pitch in question by 4/3 [because P + P(1/3) = P(3/3) + P(1/3) = P(4/3)]. Therefore, to find the value for *Tài Cù* we can simply multiply the value of *Lín Zhōng* by 4/3, to get 8/9 [because 2/3 × 4/3 = 8/9]. The rationale is similar for a down-generation: we simply need to multiply the value of the previous pitch by 2/3 [because P – P(1/3) = P(3/3) – P(1/3) = P(2/3)].

Figure 23 shows a graphic representation of the twelve pitches generated by the *sanfen sunyi* method (or up-and-down principle), as calculated by C.-Y. Chen. Notice that Chen's values are the same as Needham and Robinson's but that Chen arrives at these values by multiplying by 2/3 (for a down-generation) and by 4/3 (for an up-generation). For, as Chen explains:

> The explicit harmonic ratios used are the perfect fifth 2/3 and the perfect octave ½.... The perfect fifth interval is used to generate all the notes, while the octave is used, when needed, to regulate the generated notes so that they all lie within an octave. Thus, the up-and-down principle insures that the notes generated by the fifth are lying within the compass of an octave. [26]

26 C. Y. Chen, *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and*

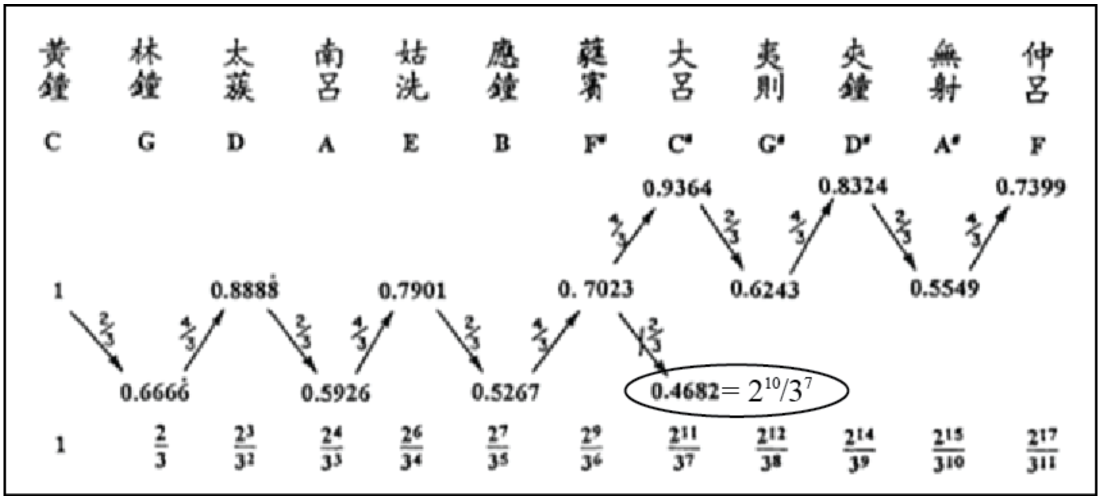

Figure 23. The generation of the *shí'èr lǜ* using the intervals of the perfect fifth and the perfect octave, based on the up-and-down principle described in the *Lǚshì chūnqiū*. (Figure 19 from C.-Y. Chen, *Early Chinese Work in Natural Science*, 81)

Table 4. *Sanfen sunyi* Pitches, Chromatic Order

Line	Pitches in Chromatic Order	Chen's Values	Re-written as $(2/3)^X \times (2)^Y$	Decimal Value
1	C *Huáng Zhōng*	Fundamental		1.000
2	C# *Dà Lǚ*	$2^{11} / 3^7$	$(2/3)^7 \times 2^4$	.9364
3	D *Tài Cù*	$2^3 / 3^2$	$(2/3)^2 \times 2^1$	.8889
4	D# *Jiá Zhōng*	$2^{14} / 3^9$	$(2/3)^9 \times 2^5$	.8324
5	E *Gū Xiǎn*	$2^6 / 3^4$	$(2/3)^4 \times 2^2$	.7901
6	F *Zhòng Lǚ*	$2^{17} / 3^{11}$	$(2/3)^{11} \times 2^6$	.7399
7	F# *Ruí Bīn*	$2^9 / 3^6$	$(2/3)^6 \times 2^3$	.7023
8	G *Lín Zhōng*	2/3	$(2/3)^1 \times 2^0$	.6667
9	G# *Yí Zé*	$2^{12} / 3^8$	$(2/3)^8 \times 2^4$	.6243
10	A *Nán Lǚ*	$2^4 / 3^3$	$(2/3)^3 \times 2^1$	.5926
11	A# *Wú Yì*	$2^{15} / 3^{10}$	$(2/3)^{10} \times 2^5$	.5549
12	B *Yìng Zhōng*	$2^7 / 3^5$	$(2/3)^5 \times 2^2$	.5267

Scientific Thoughts (Hong Kong University Press, 1996), 50.

Table 4 gives Chen's values from Figure 23 (which match Needham and Robinson's), but arranges the notes in chromatic order, as they are in the second part of the *Lǚshì chūnqiū* text:

Chen's values, given in column 3, combine the multiplications by 2/3 and those by 2 into one fraction. Column 4 separates these values so that the multiplications by 2/3 (i.e., the fifth number: "X") and the multiplications by 2 (i.e., the number of doublings: "Y") can be seen separately. Notice the red exponents in column 4 that describe the number of doublings. Reading from the top, we see a sequence of seven numbers (4,1,5,2,6,3,0) and then a partial repetition of this sequence (4,1,5,2). As we'll now see, the 0 in this sequence can be replaced with a 7 — creating an inversion of the sequence 1,5,2,6,3,7,4 — the sequence previously noted in Table 2 and Table 3.

6. CHANGING 0 TO 7: THE SPIRAL OF FIFTHS

Needham/Robinson and Chen agree on the string length values given in Table 4. However, they disagree on the value for C, the octave of the fundamental. Chen believes that the Chinese knew the value of an exact octave: 1/2 (or .5).[27] Needham and Robinson, however, not finding any textual evidence for ancient Chinese knowledge of the true octave, believe that the octave was generated by continuing to apply the *sanfen sunyi* method. In other words, they believe that a down-generation was made from the last note referred to in the *Lǚshì chūnqiū* (*Zhòng-lǚ*: F = $2^{17}/3^{11}$), the resulting pitch being a C, an octave higher (because C is, in fact, the fifth of F).

Needham and Robinson's value for the octave is therefore $2^{17}/3^{11} \times 2/3 = 2^{18}/3^{12} = (2/3)^{12} \times 2^6$, or 262144/531441 (Figure 22) — which equals .49327. The discrepancy between Chen's and Needham/Robinson's values is what is called, in the West, the "Pythagorean comma" — which has a value of 1.01364 (calculated, here, by dividing .5 by .49327). The Pythagorean comma represents the difference between the true octave (generated by halving the string length of the fundamental) and the pitch arrived at by generating the twelfth fifth and doubling its string length six times.

The *Lǚshì chūnqiū* does, in fact, imply that the Pythagorean comma was known to the Chinese. For the fundamental, *Huáng Zhōng*, is "obtained by up-generation." In other words, the text implies the existence of a pitch that *precedes* the *Huáng Zhōng*. This pitch must have 3/4 of the string length of the *Huáng Zhōng*, because the *Huáng Zhōng* is obtained from this pitch by up-generation (i.e., by multiplication by 4/3), and $3/4 \times 4/3 = 1$. Therefore, this unnamed pitch, having a string length of 3/4, must be the note that is called, in the West, the "fourth."[28] Specifically, if the *Huáng Zhōng* is C, this note must be F. Yet there is another pitch that Needham/Robinson and Chen identify as F: the eleventh fifth (*Zhòng Lǚ*), which has a string length of $2^{17}/3^{11}$. The difference between these two F values is the Pythagorean comma: $(3/4)/(2^{17}/3^{11}) = 1.01364$.

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27 Chen, *Early Chinese Work in Natural Science*, 79–81.

28 The interval between an up-generated fifth and the note from which it is generated is called, in the West, a "fourth." This nomenclature has to do with the structure of the major scale. For example, the note F, from which the note C is up-generated, is the fourth note in the scale of C Major: C,D,E,F,G,A,B,C. It is for this reason that Needham and Robinson, in the quotation cited earlier (see footnote 1), refer to "an alternating series of fifths and fourths."

Although Chen agrees that the Pythagorean comma was known to the Chinese, he believes that the exact octave was used to cap scales. Nevertheless, we will use Needham and Robinson's value for the octave. This is not meant to dismiss Chen's claim that the ancient Chinese had knowledge of the true octave. Rather, it is to identify the *shí'èr lǜ* as generated from what, today, is called the "spiral of fifths." Consequently, the resulting scale does not contain the perfect octave.

We therefore use Needham and Robinson's value for the octave in line 13 of Table 5 (lines 1 to 12 of Table 5 duplicate Table 4). Notice that the Y exponent of this value is 6 and that, as a result, the sequence already noted (in lines 2 to 8) — 4,1,5,2,6,3,0 — continues to be generated.

Table 5. Extending the Twelve-tone Scale Generated by the *Sanfen sunyi* Method Upward

Line	Pitches in Chromatic Order	Quantification	Re-written as $(2/3)^X \times (2)^Y$
1	C		Fundamental
2	C#	$2^{11} / 3^7$	$(2/3)^7 \times 2^4$
3	D	$2^3 / 3^2$	$(2/3)^2 \times 2^1$
4	D#	$2^{14} / 3^9$	$(2/3)^9 \times 2^5$
5	E	$2^6 / 3^4$	$(2/3)^4 \times 2^2$
6	F	$2^{17} / 3^{11}$	$(2/3)^{11} \times 2^6$
7	F#	$2^9 / 3^6$	$(2/3)^6 \times 2^3$
8	G	$2/3$	$(2/3)^1 \times 2^0$
9	G#	$2^{12} / 3^8$	$(2/3)^8 \times 2^4$
10	A	$2^4 / 3^3$	$(2/3)^3 \times 2^1$
11	A#	$2^{15} / 3^{10}$	$(2/3)^{10} \times 2^5$
12	B	$2^7 / 3^5$	$(2/3)^5 \times 2^2$
13	C (Needham/Robinson)	$2^{18} / 3^{12}$	$(2/3)^{12} \times 2^6$
14	C# (Chen)	$2^{10} / 3^7$	$(2/3)^7 \times 2^3$
15	D (Author)	$2^2 / 3^2$	$(2/3)^2 \times 2^0$
16	D# (Author)	$2^{13} / 3^9$	$(2/3)^9 \times 2^4$
17	E (Author)	$2^7 / 3^4$	$(2/3)^4 \times 2^1$
18	F (Author)	$2^{16} / 3^{11}$	$(2/3)^{11} \times 2^5$

Line	Pitches in Chromatic Order	Quantification	Re-written as $(2/3)^X \times (2)^Y$
19	F# (Author)	$2^8 / 3^6$	$(2/3)^6 \times 2^2$

We now continue up the scale, as shown in Table 5. The value for the C# in line 14 has already been calculated by Chen in Figure 23 as .4682, or $(2/3)^7 \times 2^3$. It makes sense that this value is the same as the value given for the C# in line 2, but with a Y exponent of 3 rather than 4, because this C# is in the next higher octave, so its string length is 1/2 that of the C# in line 2. Using this rationale, it's easy to calculate the string lengths of the pitches in lines 15 to 19 of Table 5: each note's string length will be given by an expression that is similar to the one for the note of the same name in the lower octave, but will have a Y exponent that is 1 number less. For example, D in line 3 has a value of $(2/3)^2 \times 2^1$, so D in line 15 will have a value of $(2/3)^2 \times 2^0$.

We now have the sequence 4,1,5,2,6,3,0 repeating almost three times. Let's see if it's possible to change the 0s in lines 8 and 15 to 7s and, in so doing, have the sequence generated by the up-and-down principle be an inversion of the sequence 1,5,2,6,3,7,4 — which we saw earlier to be written on cuneiform tablet CBS 1766 and embedded in the instructions in the *Lǚshì chūnqiū*.

To change the 0 in line 8 to a 7 we take Needham and Robinson's C in line 13 of Table 5 and apply an up-generation: $(2/3^{12} \times 2^6) \times (4/3) = 2/3^{13} \times 2^7$. In so doing, we are re-calculating the G in the first octave as its Pythagorean-comma-equivalent (line 8 of Table 6). Notice that, in so doing, the Y exponent has been increased (from the value in line 8 of Table 5) by a value of 7, and the X exponent has been increased by a value of 12.

Table 6. Changing the 0s to 7s

Line	Pitches in Chromatic Order	Quantification	Written as $(2/3)^X \times (2)^Y$
1	C	Fundamental	
2	C#	$2^{11} / 3^7$	$(2/3)^7 \times 2^4$
3	D	$2^3 / 3^2$	$(2/3)^2 \times 2^1$
4	D#	$2^{14} / 3^9$	$(2/3)^9 \times 2^5$
5	E	$2^6 / 3^4$	$(2/3)^4 \times 2^2$
6	F	$2^{17} / 3^{11}$	$(2/3)^{11} \times 2^6$
7	F#	$2^9 / 3^6$	$(2/3)^6 \times 2^3$
8	G (Author)	$2^{20} / 3^{13}$	$(2/3)^{13} \times 2^7$
9	G#	$2^{12} / 3^8$	$(2/3)^8 \times 2^4$
10	A	$2^4 / 3^3$	$(2/3)^3 \times 2^1$
11	A#	$2^{15} / 3^{10}$	$(2/3)^{10} \times 2^5$
12	B	$2^7 / 3^5$	$(2/3)^5 \times 2^2$
13	C (Needham/Robinson)	$2^{18} / 3^{12}$	$(2/3)^{12} \times 2^6$
14	C# (Chen)	$2^{10} / 3^7$	$(2/3)^7 \times 2^3$
15	D (Author)	$2^{21} / 3^{14}$	$(2/3)^{14} \times 2^7$
16	D# (Author)	$2^{13} / 3^9$	$(2/3)^9 \times 2^4$
17	E (Author)	$2^7 / 3^4$	$(2/3)^4 \times 2^1$
18	F (Author)	$2^{16} / 3^{11}$	$(2/3)^{11} \times 2^5$
19	F# (Author)	$2^8 / 3^6$	$(2/3)^6 \times 2^2$
20	G (Author)	$2^{19} / 3^{13}$	$(2/3)^{13} \times 2^6$

Now, to change the 0 in line 15 to a 7 we apply two steps. First, we start, as before, with Needham and Robinson's C in line 13, but this time we apply a down-generation. The resulting note is a G, with a string length value of $(2/3^{12} \times 2^6) \times (2/3) = 2/3^{13} \times 2^6$. This G (line 20, Table 6) is in the second octave. We now apply an up-generation to this G. In so doing, we are re-calculating the D in line 15 as its

Pythagorean-comma-equivalent: $(2/3^{13} \times 2^{6}) \times (4/3) = 2/3^{14} \times 2^{7}$. As before, the Y exponent of this note has been increased (from Table 5) by 7 and the X exponent by 12.

These numbers — 7 and 12 — describe a fundamental truth about the relationship between the fractions 1/2 and 2/3. For when 2/3 is multiplied by itself 12 times the resulting number is *almost the same* as that generated by multiplying 1/2 by itself 7 times ($2/3^{12} \approx 1/2^{7}$), the difference being the Pythagorean comma.

What this means, musically, is that if twelve consecutive fifths are generated (the equivalent of multiplying 2/3 by itself 12 times) the resulting pitch is *almost* (except for the Pythagorean comma) the same as the seventh higher octave of the fundamental (the equivalent of multiplying 1/2 by itself 7 times). This fact establishes a general rule: two notes that are removed from each other by twelve positions in a continuous line of fifths will be almost the same note (the difference being the Pythagorean comma), but will be separated by seven octaves.

This explains the nature of the Y exponent 7: it is generated when, instead of using a fifth that is located in the octave of the scale under construction (i.e., that requires 0 doublings), we use a fifth that is twelve positions higher. The string length of this higher fifth must then be doubled 7 times, in order to find this same note in the octave of the scale that is under construction.

In discarding the fifths that lie within the octave under construction and using, instead, their higher counterparts (i.e., in replacing the 0s with 7s), we are now applying the same rule to create all the notes in the scale: *every note is now generated by doubling the string length of a fifth that lies above the octave under construction*. Because the string length of a fifth is always doubled between 1 and 7 times, the sequence 4,1,5,2,6,3,7 is generated (column 4, Table 6). Moreover, because there are an infinite number of octaves and fifths, the sequence will repeat ad infinitum.

Figure 24 shows the string lengths from Table 6, when measured relative to a fundamental that is 60 cm in length — the approximate length of a guitar string. For example, the green line shows the length of string required to sound the note G#: it is the value given in line 9 of Table 6 [(G#,8, 4) = $(2/3)^{8} \times 2^{4}$ = .6243], multiplied by 60 cm: 37.458 cm. If we plot the string lengths of all the pitches in Table 6, vertically, we create the template of the guitar fretboard in Figure 24. Relative to an actual guitar fretboard the positions of the frets are slightly altered because they are generated from perfect fifths (i.e., Pythagorean Tuning) rather than by modern, equal temperament.

It is important to note that the diagram in Figure 24 can be created without measuring, simply by folding a strip of paper.[29] First, one end of the paper is labelled "bridge." Then, the paper is folded in thirds and the fold that is 2/3 of the distance from the bridge is marked with a line, or "fret." The length between the bridge and this fret is then folded in thirds, and, again, the 2/3 position is marked with a fret. This second fret will lie between the first octave (the red line) and the bridge. Consequently, this string length must be doubled and another fret drawn to show the location of this same note in the next lower octave. Repeating this process derives all the frets in Figure 24. Keeping a tally of the number of 2/3 folds gives the X exponents in column 4 of Table 6; keeping a tally of the number of doublings gives the Y exponents: the sequence 4,1,5,2,6,3,7.

29 A description of this exercise is available online at: http://musicircle.net/paper-folding-exercise/

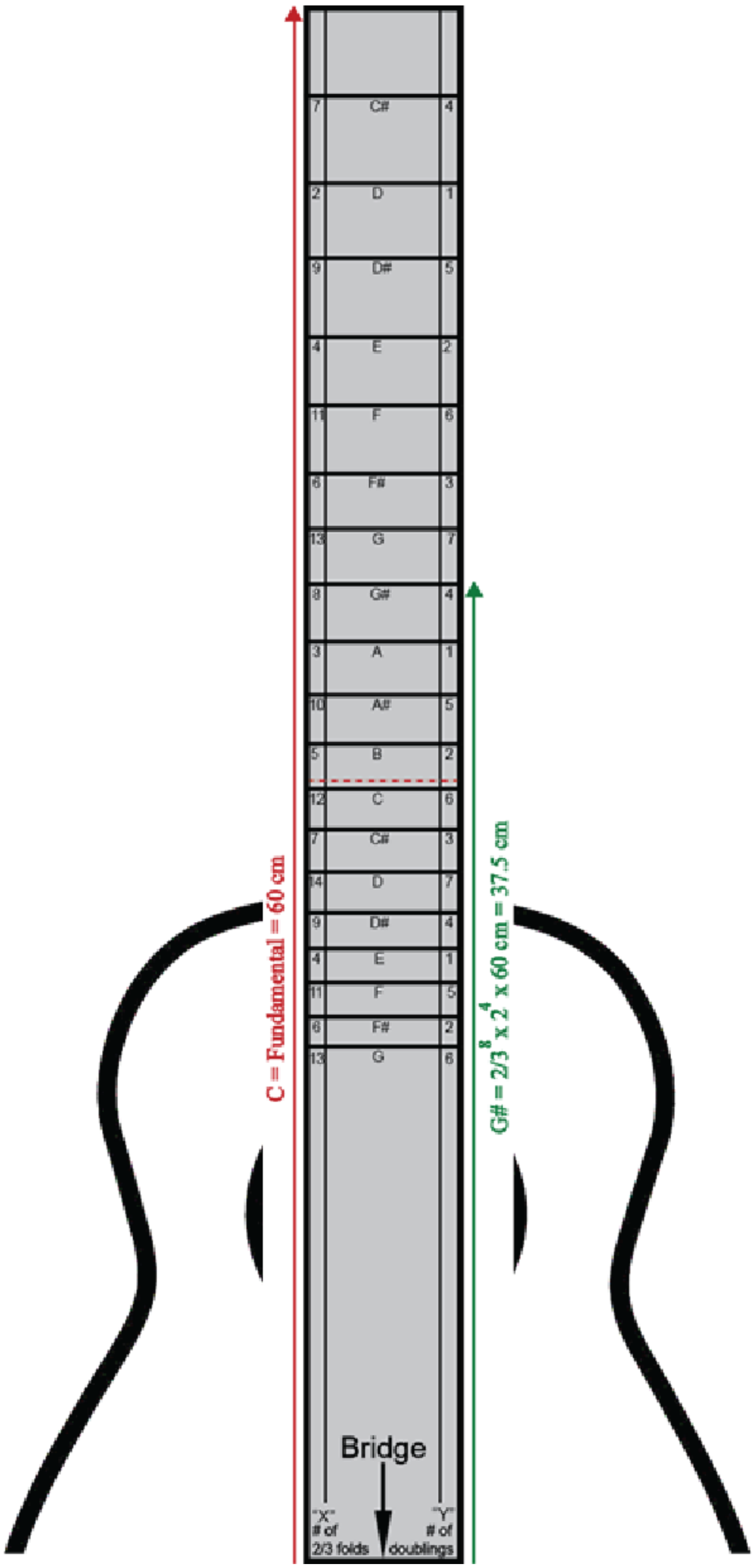

Figure 24. Guitar Fretboard, Generated Using the Ancient Chinese *Sanfen sunyi* Method (not to scale)

What we can note from this paper-folding exercise is that the data in column 4 of Table 6 can be derived without doing any mathematics, but simply by keeping a tally of the number of 2/3 folds and the number of doubling folds generated by the instructions in the *Lǚshì chūnqiū*. In other words, the up-and-down principle can be illustrated by a simple, hands-on exercise that could have been done, in ancient times, with whatever materials were available: leather, fabric, string, etc.

In conclusion, we have seen that the sequence 4,1,5,2,6,3,7 — an inversion of the previously noted sequence 1,5,2,6,3,7,4 — is generated when creating a twelve-tone scale from the spiral of fifths using the ancient Chinese *sanfen sunyi* method. We have also seen that inversions of this sequence are written on cuneiform tablet CBS 1766 (and, as we will soon see, on tablet UET VII 74). What will be shown, now, is that Figure 24 — the linear diagram that shows the twelve-tone scale generated by the ancient Chinese *sanfen sunyi* method — can be converted into a circular diagram that resembles the drawing on CBS 1766.

7. DERIVING A LIKENESS OF THE DIAGRAM ON CBS 1766

To begin our derivation, we arrange the notes as they are listed as consecutive fifths in the *Lǚshì chūnqiū*: C, G, D, A, E, B, F#, C#, G#, D#, A#, F. Now, because the 12th fifth is, essentially, another C, the list can be written as a circle — what, today, we call the *circle of fifths* (Figure 25).

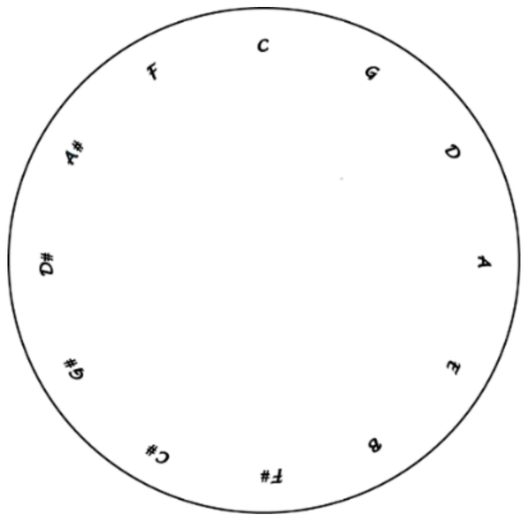

Figure 25. Circle of Fifths

Now, let's look at the relationship between the circle of fifths and the chromatic order of notes given later in the *Lǚshì chūnqiū* text: C, C#, D, D#, E, F, F#, G, G#, A, A#, B. As shown in Figure 26, relative to the circle of fifths, the note C# is seven positions clockwise from the note C. The pink line in Figure 26 shows this relationship, illustrating the ascent of one semitone in the chromatic scale. Similarly, D, which is one semitone above C#, is seven positions clockwise from C#, as shown by the orange line. Continuing by increments of one semitone, we draw a line that leads, each time, seven positions clockwise. The result is the twelve-pointed star in Figure 26. [30]

30 Printable templates of the models used in this paper are available at: http://musicircle.net/printable-templates/

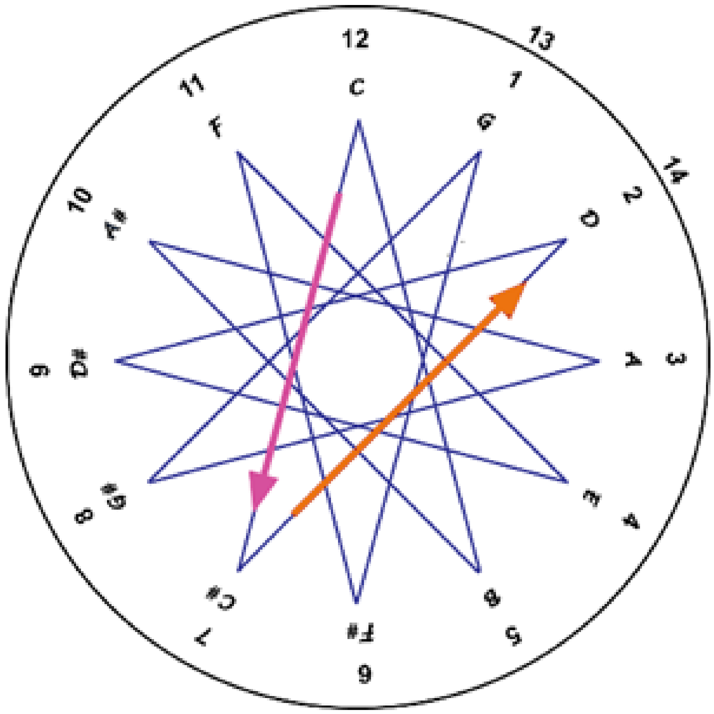

Figure 26. Twelve-Pointed Star Showing the Relationship between the Circle of Fifths and the Chromatic Scale

In Figure 26, the number of each consecutive fifth (i.e., the X exponents from column 4, Table 5) is written inside the circle, in line with the pitch to which it corresponds. The X exponents for the notes G and D from lines 8 and 15 of Table 6 are also written in Figure 26, *outside* the circle. The addition of these two numbers will allow us to begin generating the *spiral of fifths*.

To derive a heptagram similar to the one on CBS 1766, we simply transfer the Y exponents from column 4 of Table 5 (printed in red) onto the points of the star, as shown in Figure 27. We begin with the note in line 2 of Table 5, C#, and simply copy its Y exponent — the number 4 — onto the point of the star that indicates C#. In this way, the data from line 2 of Table 5 (C#, 7, 4) is now duplicated on the diagram in Figure 27: the X exponent, 7, is written radially outward from the note C# and the Y exponent, 4, is written on the associated point of the star.

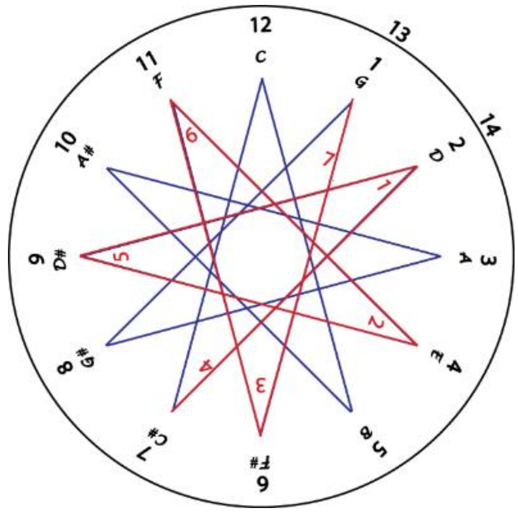

Figure 27. Transferring the Y Numbers

To ascend one step up the chromatic scale to the next note, D (line 3 of Table 5), we travel along the diagonal of the star, *highlighting it in red*. Now we write the number 1 on this point of the star, thereby copying the data from line 3 of Table 5 (D, 2, 1) onto our diagram. We now move along the diagonal to the next note in the scale, *again highlighting it in red*. This brings us to D#, and we write 5 on this point of the star, thereby duplicating the data from line 4 of Table 5 (D#, 9, 5). We continue along the diagonals, highlighting them and copying the Y exponents onto the points of the star. We continue until we have transferred the data from line 7 (F#, 6, 3).

We have now arrived at the first 0 in Table 5. Remember, however, that we changed this 0 to a 7 *by generating all the notes in the scale from fifths that lie* <u>*above*</u> *the octave under construction*. Consequently, we now move to line 8 of Table 6, which gives the X exponent of G as 13 and the Y exponent as 7: (G,13, 7). This agrees with our diagram *if we use the X number written outside the circle*. (We'll see that this is a general rule: in order for our diagram to give the correct X exponents as we ascend the scale, each time we arrive at a note with a Y exponent of 7 we must add 12 to its X exponent.) As before, we write 7 on the point of the star that indicates G.

We have now completed one cycle of the sequence 4,1,5,2,6,3,7. Notice that copying the next seven Y exponents from Table 6 onto our star will simply reproduce this sequence. This gives us an idea: what if, instead of copying more numbers, *we simply rotate our construction one position clockwise so that the point labelled 4 indicates G#, the next note in the scale*? (Figure 28)

In doing this, our model agrees with line 9 of Table 6: (G#, 8, 4), but will our model continue to agree with Table 6 as we ascend the scale? Moving along the diagonal to the next note, A, our model gives (A, 3, 1) and this does, in fact, match line 10 of Table 6. And as we ascend the scale, our model and Table 6 continue to correspond: (A#, 10, 5), (B, 5, 2), (C, 12, 6), (C#, 7, 3), (D, 14, 7). (D has a Y exponent of 7, so we add 12 to its X exponent: 2+12 = 14.)

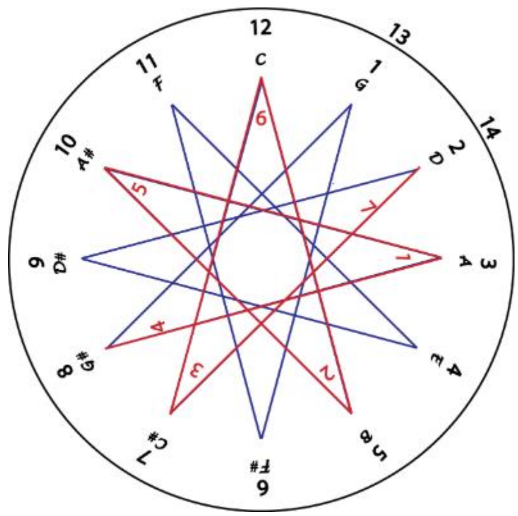

Figure 28. Our Construction, Rotated

If we were to continue up the scale, we would see the above pattern repeat: whenever a note with a Y exponent of 7 is reached, we must add 12 to its X exponent. We then turn our construction 30° clockwise and continue our ascent.

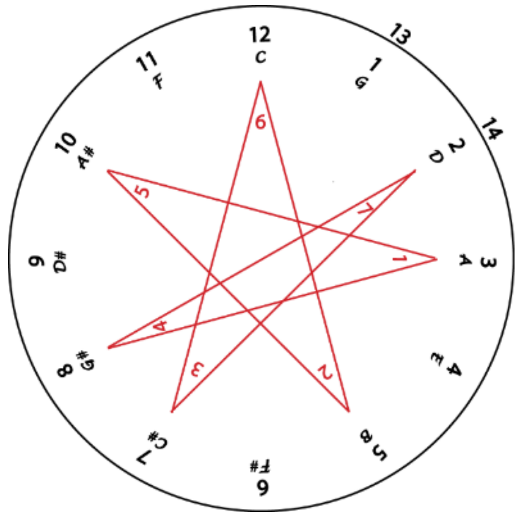

Figure 29. A Heptagram

We see, therefore, that because there are only seven Y exponents, and because the rotation of the star allows us to pair these numbers correctly (i.e., as they are given in Table 6) with the spiral of fifths, we have no need for the five unused points of the star, *so we can remove them*. We then draw a line, joining point 7 and point 4. The result is the heptagram shown in Figure 29.

Incidentally, our model can also be used to generate a *descending* scale, by rotating the heptagram *counterclockwise* and *subtracting* 12 from every seventh X exponent. The resulting values are shown in Table 7. As we might expect, to generate the scale in the next lower octave, the Y exponent of most notes will be increased by 1. (For example, the D# in line 4 of Table 6 — shown in Table 7 as line 16 — has a value of $(2/3)^9 \times 2^5$, so the D# in line 4 of Table 7 has a value of $(2/3)^9 \times 2^6$.) When a note has a Y exponent of 7, however, to find the value of this note in the next lower octave, we change its Y exponent to 1 and subtract 12 from its X exponent. For example, G in line 20, Table 7, has a value of $(2/3)^{13} \times 2^7$, so G in line 8 has a value of $(2/3)^1 \times 2^1$.

Table 7. Extending the Scale Downward

Line	Pitches in Chromatic Order	$(2/3)^X \times (2)^Y$	Fraction Equivalent
1	C (Fundamental)	$(2/3)^0 \times 2^1$	2
2	C#	$(2/3)^7 \times 2^5$	4096/2187
3	D	$(2/3)^2 \times 2^2$	16/9
4	D#	$(2/3)^9 \times 2^6$	32768/19683
5	E	$(2/3)^4 \times 2^3$	128/81
6	F	$(2/3)^{11} \times 2^7$	262144/177147
7	F#	$(2/3)^6 \times 2^4$	1024/729
8	G	$(2/3)^1 \times 2^1$	4/3
9	G#	$(2/3)^8 \times 2^5$	8192/6561
10	A	$(2/3)^3 \times 2^2$	32/27
11	A#	$(2/3)^{10} \times 2^6$	65536/59049
12	B	$(2/3)^5 \times 2^3$	256/243
13	C	$(2/3)^{12} \times 2^7$	524288/531441
14	C#	$(2/3)^7 \times 2^4$	2048/2187
15	D	$(2/3)^2 \times 2^1$	8/9
16	D#	$(2/3)^9 \times 2^5$	16384/19683
17	E	$(2/3)^4 \times 2^2$	64/81
18	F	$(2/3)^{11} \times 2^6$	131072/177147
19	F#	$(2/3)^6 \times 2^3$	512/729
20	G	$(2/3)^{13} \times 2^7$	1048576/1594323
21	G#	$(2/3)^8 \times 2^4$	4096/6561
22	A	$(2/3)^3 \times 2^1$	16/27
23	A#	$(2/3)^{10} \times 2^5$	32768/59049
24	B	$(2/3)^5 \times 2^2$	128/243
25	C	$(2/3)^{12} \times 2^6$	262144/531441

Applying these rules, we can extend our scale downward indefinitely. However, in the next lower octave we'll start to generate negative X exponents — and negative numbers are a modern concept. Therefore, let's take the note C in line 1 of Table 7 to be the natural lowest extent of our scale, especially since the string length of this note (our new fundamental) is a whole number: 2.[31]

Notice that, in line 1 of column 3 of Table 7, the sequence now starts with the number 1: 1,5,2,6,3,7,4 — and that this is the same sequence embedded in the instructions in the *Lǚshì chūnqiū*. As we'll see shortly, this exercise of generating the scale in a descending direction will have an application when we look at the Mesopotamian tonal system.

Let's summarize. The model we've made is a simple "computer" that calculates the string lengths of the notes in a twelve-tone scale generated by the ancient Chinese *sanfen sunyi* method. To recap the procedure: the fundamental is multiplied by 2/3 X number of times to generate a specific fifth. The string length of that fifth is then doubled between 1 and 7 times to locate that same note in the octave under construction. This process is represented by the formula $2/3^{X} \times 2^{Y}$. Consequently, to use our model to calculate the string lengths of the notes in the scale, we simply read off the X and Y exponents from our model and input them into this formula.[32]

31 In fact, giving the whole string a value of 2 is standard practice in physics, because the wavelength that sounds the fundamental (the lowest resonant frequency) is twice the length (L) of the string: $\lambda = 2L$, where λ is "wavelength."

32 According to Mathieu Ossendrijver, Professor of History of Ancient Science at Humboldt University, there are no conceptual difficulties for an Old Babylonian scholar in the computations described here. For keep in mind that exponents are simply a means of expressing the number of times that a specific number is multiplied by itself.

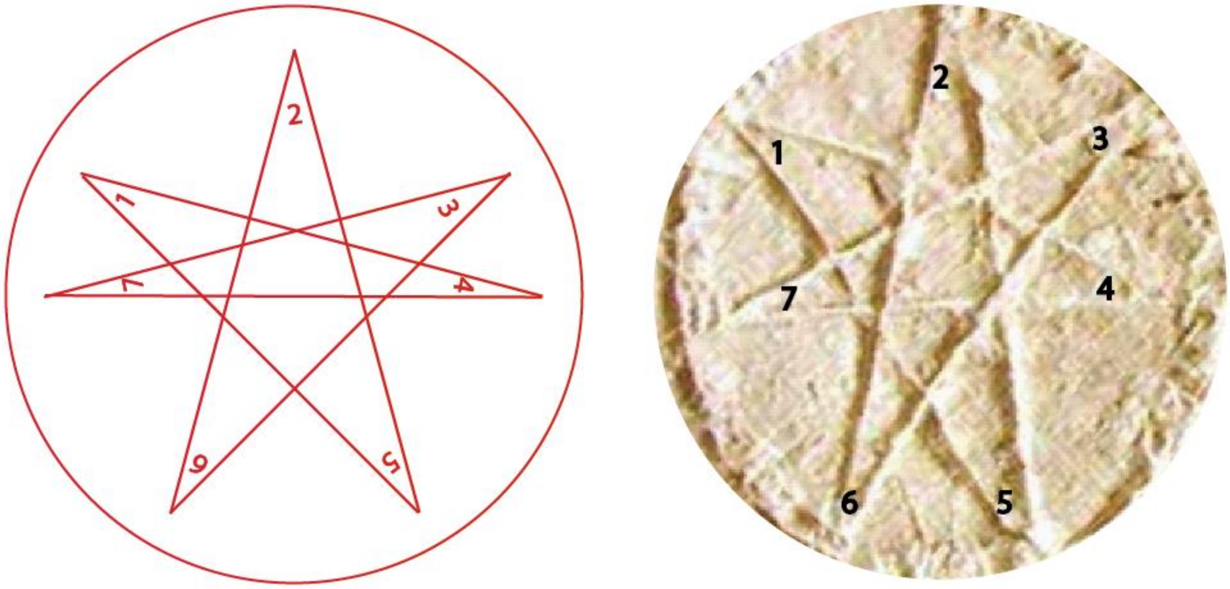

Figure 30. Comparing the Derived Heptagram with the Heptagram on CBS 1766

Figure 30 shows the heptagram from Figure 29, rotated so that the point numbered 2 is at the 12 o'clock position. Also shown in Figure 30 is the diagram on CBS 1766, rotated so that the point of the heptagram labelled 2, in cuneiform script, is also at the 12 o'clock position. Notice the likeness between the two heptagrams: both have similarly spaced points,[33] identically numbered.

But are the two heptagrams, in fact, the same? Is the diagram on CBS 1766 related to a twelve-tone scale generated by the *sanfen sunyi* method? To answer these questions, let's look more closely at the tonal systems of both Mesopotamia and ancient China.

33 CBS 1766 is believed to be the schoolwork of a student, and the diagram is thought to have been drawn freehand.

8. THE ANCIENT CHINESE TONAL SYSTEM: MUSICAL INSTRUMENTS OF EARLY CHINA

There are two types of sources that throw light on the tonal system of ancient China: texts and artifacts. The earliest textual description of the generation of a scale is found in the *Guanzi* (管子), a compilation of anonymous essays dating between the fifth century BC and the first century BC. We will look at this text shortly, but first let's examine some of the musical artifacts.

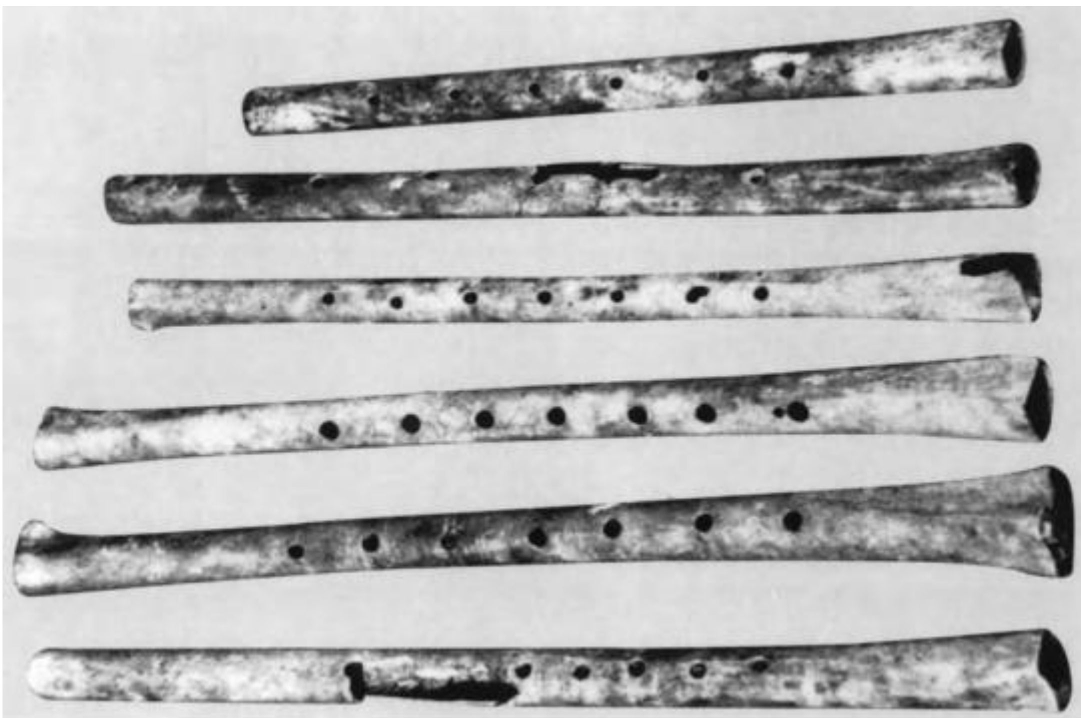

Figure 31. Bone flutes from Henan. Seventh millennium BC (Henan Museum, China)

The oldest musical instruments discovered in China are longitudinal bone flutes, some of which date to the seventh millennium BC (Figure 31). However, these instruments do not give reliable pitch measurements and therefore provide no definite information about musical scales.

The oldest instruments with pitches that can still be sounded are bronze bells. Moreover, sets of bells used as a single musical instrument — known as *bianzhōng* (編鐘) (Figure 32) — have provided information regarding the nature of the scales used by the ancient Chinese.

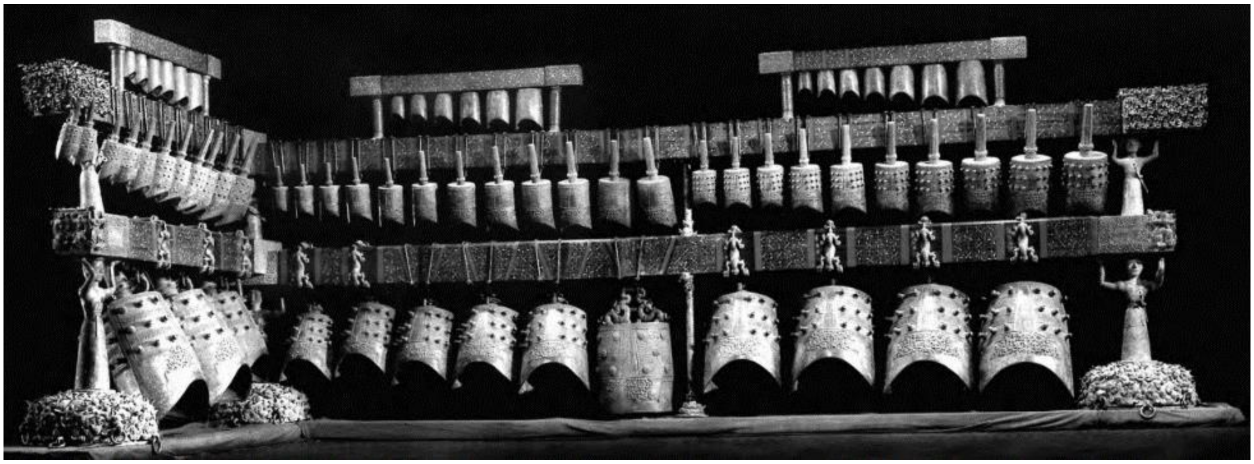

Figure 32. *Bianzhōng*: sixty-five bronze bell set. Tomb of Marquis Yi, 433 BC (Hubei Provincial Museum, China)

The *bianzhōng* evolved over centuries.The earliest bronze bells discovered in China, dating to circa 1700 BC, were not made for musical purposes at all. Instead, these clapper bells were probably used as signalling devices. Figure 33 shows a clapper bell found in the province of Henan at Erlitou (see map: Figure 34), an archaeological site containing China's earliest bronze artifacts, some dating to 1900 BC. Later, single bells began to be used as musical accompaniment.

R. Bagley suggests that by 1200 BC individual bells were being used to accompany music,[34] for one of the first known sets of bells (that is thought to have been collected, rather than intentionally cast as an instrument) is dated to the eleventh century BC.

This set, unearthed in 1993, near Changsha, in the province of Hunan (see map: Figure 34) contains ten bells, nine of which have the same exterior decoration (Figure 35). In examining the pitches of all ten bells, Bagley notes that "starting on C# and going up, their pitches include six consecutive semitone steps [C#, D, D#, E, F, and F#] of very nearly the same size.... The music master who formed this set knew the chromatic scale." [35]

34 Robert Bagley, "Ancient Chinese Bells and the Origin of the Chromatic Scale," *Zhejiang University Journal of Art and Archaeology* 2 (2015): 69.

35 Bagley, "Origin of the Chromatic Scale," 72.

Figure 33. Clapper bell (ling). Erlitou, 1800–1600 BC (Arthur M. Sackler Gallery, Washington, D. C.)

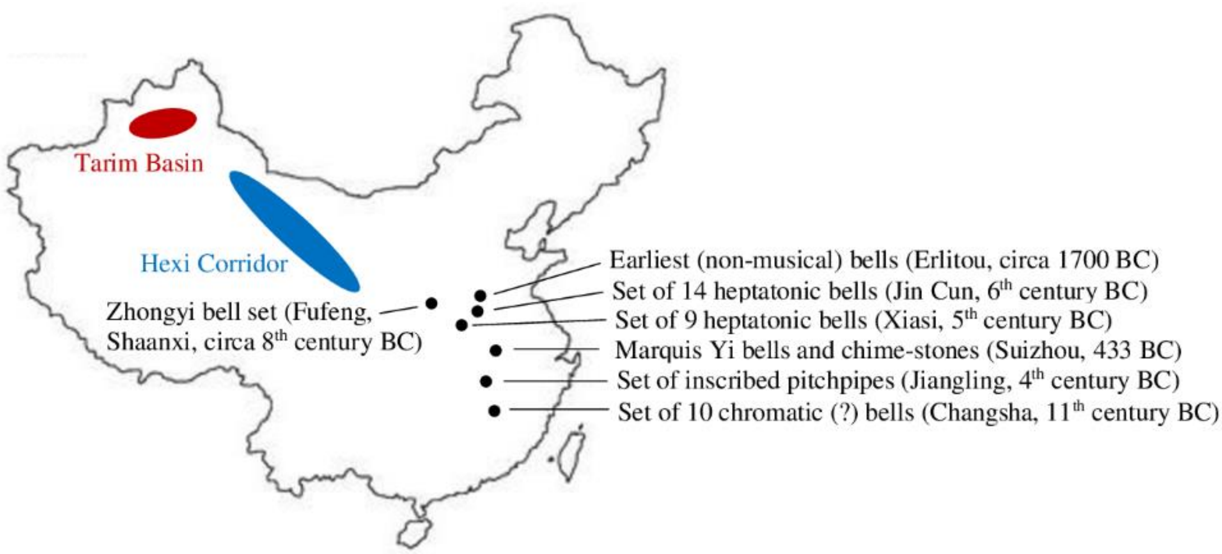

Figure 34. Map of China showing the archaeological sites where several musical artifacts were found

Bagley believes that the chromatic scale developed in China after the pentatonic scale was already in use. He suggests that the situation that brought about its development was the musician's need to transpose the pentatonic scale, in order for it to harmonize with the different pitches sounded by individual signalling bells.[36] Musically speaking, this is a reasonable hypothesis, for transposing the pentatonic scale upward or downward by three consecutive semitones does, in fact, generate the twelve *shí'èr lǜ* pitches. However, there is little evidence for the use of the pentatonic scale that predates the "chromatic" Changsha bell set, described earlier by Bagley, which dates to the eleventh century BC (i.e., to the beginning of the Early Zhou period):

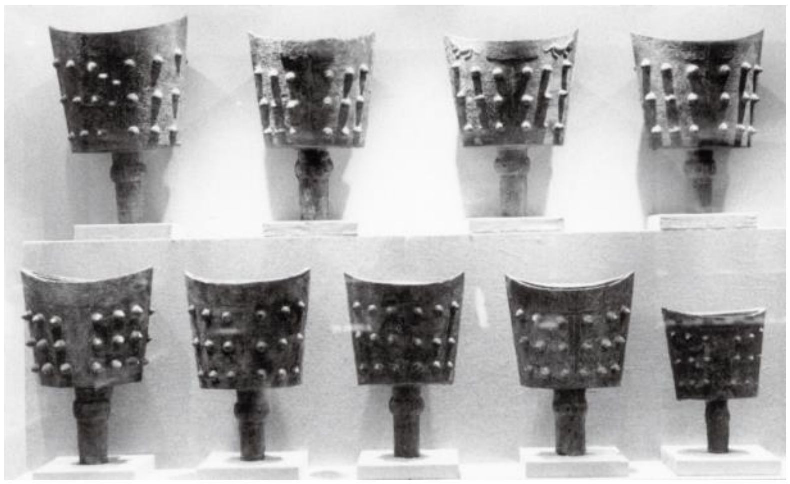

Figure 35. Nine bells from a deposit of ten found near Changsha (circa eleventh century BC). (Photo from R. Bagley "Ancient Chinese Bells and the Origin of the Chromatic Scale," Figure 24)

> Whether the pentatonic scale was the norm by the Early Zhou [1046 BC to 771 BC] is somewhat uncertain. Early Zhou bell ensembles in fact reveal a preference for the same four-tone scale as their Shang predecessors, at least when we take the alternate striking tones into account. For instance, the late-Western Zhou Zhongyi *yong* bell set from Fufeng, Shaanxi, produces on the main striking portions only three distinct tones over the span of three octaves: *la-do-mi-la-mi-la-mi-la*; but assuming the right striking portion was also intended for playing, the *mi* and *la* bells also produce *sol* and *do* notes

36 Bagley, "Origin of the Chromatic Scale," 57.

(each a minor third higher), suggesting the use of a four-note scale of tonic, major third, perfect fifth, major sixth (*do-mi-sol-la*).[37]

As described in the above quotation, ancient Chinese bells were capable of playing two distinct pitches. One of these pitches was produced by striking the front of the bell, the other by striking its side. This unique acoustic feature is due to the almond-shaped cross section of the bells. On the bells from the tomb of Marquis Yi of Zeng, the two strike-portions are inscribed with the names of the two tones they produce (Figure 36), indicating that the practice of sounding two tones from a single bell was definitely in use by the fifth century BC.

Figure 36. Almond-shaped mouth of a Marquis Yi of Zeng bell, with main strike-portion labelled *gōng*. (Photo from R. Bagley, "Ancient Chinese Bells and the Origin of the Chromatic Scale," Figure 17)

According to the above quotation, in the Early (or Western) Zhou period (1045–771 BC) bell sets give no clear indication of the scale(s) in use: the bell set from Fufeng (see map: Figure 34) could have played a three-note scale (*do-mi-la*) or a four-note scale (*do-mi-sol-la*) — or perhaps these bells were used to emphasize specific pitches in a scale having more than four notes.

According to the same source, the *Oxford Handbook of Early China*, from the eighth century BC onward sets of bells appear to suggest the use of the pentatonic scale:

37 Scott Cook, "Music in the Zhou," *Oxford Handbook of Early China* (Oxford University Press, 2020), 480.

> By the time of the Chunqiu period [771–476 BC], however, the pentatonic scale (including the major second … or *re*), appears to predominate in bells and chime-stones as well, as evidenced perhaps from the Shangmacun chimestone set.... Pitch testing on some of the larger *niu*-bell sets also tends to suggest the use of the standard major pentatonic scales on the main striking portions … with the possibility of even producing heptatonic scales with major sevenths and raised fourths once the alternate striking portions are taken into account.[38]

As stated above, the Shangmacun chimestone set *may* give evidence for the use of the pentatonic scale and the *niu*-bell sets *tend to suggest* the same. In other words, evidence for the pentatonic scale in the Chunqiu period is still not concrete. This is the general consensus, for as Bagley admits, "though pentatonic music may have existed in China long before the fifth century, the Zeng instruments and inscriptions are at present the earliest unambiguous evidence for it."[39]

According to the *Oxford Handbook of Early China*, the *niu*-bell sets can produce heptatonic scales if alternate strike-portions are used — and, in fact, there is perhaps more solid evidence for the heptatonic scale in the Chunqiu period. A set of fourteen bells (one is shown in Figure 37) found at Jin Cun, Henan (see map: Figure 34), and dated to the sixth century BC [40] gives a heptatonic scale. These "Biao" bells (cast to honor Duke Biao) "supply three octaves of the seven-note scale *do*, *re*, *mi*, *fa*, *sol*, *la*, *ti*, *do* (i.e., the Western major scale, known also in later Chinese music)."[41] Moreover, a nine-bell set from Xiasi, Xichuan, Henan (see map: Figure 34), also from the Chunqiu period, plays a major scale (with some inexact pitches) on the main strike-portions.[42]

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38 Cook, "Music in the Zhou," *Oxford Handbook*, 480.

39 Robert Bagley, "The Prehistory of Chinese Music Theory," *Proceedings of the British Academy* 131 (2005): 63.

40 https://collections.rom.on.ca/objects/292379/niuzhongbell

41 Bagley, "Prehistory of Chinese Music," 78.

42 Li Chunyi, "A History of Pre-Imperial Chinese Music," rev. ed. (Beijing: Renmin yinyue chubanshe, 2005), 124, example 4.

Figure 37. One of Fourteen "Biao" Bells, sixth century BC (Royal Ontario Museum, Toronto, Canada)

By far the most astounding archaeological find that throws light on the tonal system of ancient China is the set of musical instruments discovered in the tomb of Marquis Yi of Zeng. This tomb, dated to 433 BC, was unearthed in 1977 near Suizhou, in the province of Hubei (see map: Figure 34). The tomb contained not only the *qin* shown in Figure 11, but many other instruments, the most important being the *bianzhōng* set of sixty-five bronze bells shown in Figure 32 and a set of forty-one chimestones (Figure 38). One reason for the importance of these two sets is that each bell and stone is labelled with its pitch name (two pitches for each bell) but also with detailed descriptions of how that pitch relates to specific scales. The chimestones and the bells are the only instruments in the tomb that have such musical descriptions. Let's begin by looking at the stones.

Figure 38 shows a replica of the original stand on which are hung thirty-two chimestones (also replicas). This was the arrangement of chimestones on the rack when the tomb was opened. When the chimestones were not being played they were kept in three boxes, also found in the tomb. The boxes have fitted, numbered slots showing where each numbered stone goes.

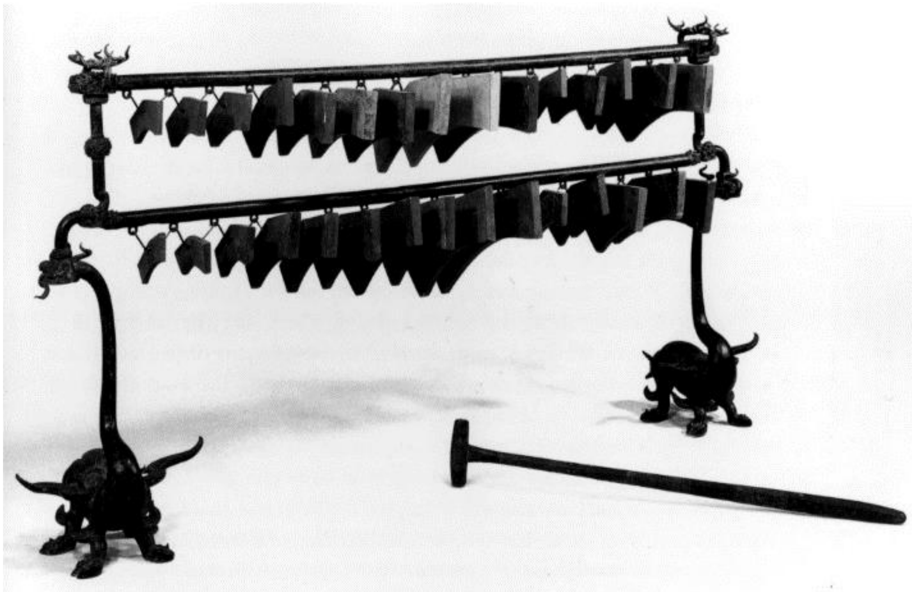

Figure 38. Replica of thirty-two chimestones. Tomb of Marquis Yi, 433 BC (Hubei Provincial Museum, China)

Box 1 contains stones that each sound one of five pitches: C, D, E, G or A (or their octaves). Box 2 contains stones that sound the pitches F#, G#, A#, C# or D# (or their octaves). In other words, the stones in Box 1 can be arranged to generate a pentatonic scale starting on C: C, D, E, G, A. Similarly, the stones in Box 2 can be arranged to generate a pentatonic scale starting on F#. Box 3 contains stones that sound the two pitches of the chromatic scale that are not found in boxes 1 and 2: the pitches B and F (and their octaves). Therefore, as Bagley describes, "the boxes ... sort the chime stones into two non-overlapping pentatonic scales plus leftovers."[43]

On each chimestone are written several inscriptions. For example, on stone no. 16 (Figure 39) a phrase is written next to the suspension hole which, to use Western pitch notation, reads "*gōng* of B." *Gōng* means the first note of the scale — the tonic — so the inscription is indicating that in the key of B this stone sounds the first note of the scale. The stone has further inscriptions:

43 Bagley, "Prehistory of Chinese Music," 63.

> On the top edge of the stone we read: "In the key of E, this is *zhǐ*." The syllable *zhǐ* means the note that lies a fifth above the tonic, and indeed the pitch of this stone, B, is a fifth above E. And the inscription continues around the edges of the stone, telling us ... where its pitch falls in each of the five pentatonic scales it belongs to. Translated … it says: "In the key of B this is *gōng*, in the key of A it is *shāng*, in the key of G it is *jué*, in the key of E it is *zhǐ*, and in the key of D, it is *yǔ*." [44]

Bagley also notes that "in addition to inscriptions locating it in five pentatonic scales, every stone in the set has inscriptions relating it to B and F#,"[45] suggesting "that every stone originally bore inscriptions relating it to five pentatonic scales and to [two] chromatic scales." [46]

This multi-labelling of each pitch — giving the pitch's position relative to five pentatonic modes and two chromatic pitches — is also found on the bells. However, with the bells there is no obvious pentatonic sorting such as is shown by the chimestones boxes. This leads Bagley to describe the bells as "elegantly, ostentatiously chromatic." [47] Yet he suggests that the pentatonic scale is, nevertheless, "embedded in the chromatic nomenclature, but not very obviously."[48] The chromatic nomenclature on the bells is as follows:

44 Bagley, "Origin of the Chromatic Scale," 65.

45 Bagley, "Prehistory of Chinese Music," 71n25.

46 Bagley, "Prehistory of Chinese Music," 71,n26.

47 Bagley, "Prehistory of Chinese Music," 68.

48 Bagley, "Prehistory of Chinese Music," 68.

Figure 39. Marquis Yi Chimestone No. 16, which sounds the pitch B (Hubei Provincial Museum)

Four core names are used: *gōng*, *shāng*, *zhǐ* and *yǔ*. (These are four of the five names — referred to earlier by Bagley — that were used to describe the notes of the pentatonic scale). To create the other eight names needed to name the twelve chromatic pitches, the suffixes *jué* and *zeng* were added to each of the four core names (Table 8). As with the Western solfege system, these names do not refer to exact pitches (C, C#, etc.). Instead, they indicate relative position (for example, *zhǐ* is the fifth note of the scale).

Table 8. Chromatic Nomenclature on the Marquis Yi Bells

Marquis Yi Bells	Western Solfege
gōng	*do*
yǔjué	*do-sharp*
shāng	*re*
zhǐzeng	*re-sharp*
gōngjué	*mi*
yǔzeng	*fa*
shāngjué	*fa-sharp*
zhǐ	*sol*
gōngzeng	*sol-sharp*
yǔ	*la*
shāngzeng	*la-sharp*
zhǐjué	*ti*

In addition to the *jué/zeng* prefixes, Zeng theorists also used the prefix *bian* to indicate the flattening of a note by one semitone (for example, *bianzhǐ* is the equivalent of *sol*-flat). As Bagley notes, "this flatting prefix hints that a scale containing some semitone steps, perhaps one of the heptatonic scales we know from later Chinese music, was in use alongside the pentatonic…."[49]

Similar to the modern Western naming system (C, C#, etc.), the pitches sounded by the bells were also given names. Yet only six pitches of the Marquis Yi bells were named: *Huáng Zhōng*, *Tài Cù*, *Gū Xiǎn*, *Ruí Bīn*, *Wei Yin* and *Wú Yì*. (Notice that five of these names are the same as those given to every second pitch listed in the *Lǚshì chūnqiū* [see column 1 of Table 1]: *Huáng Zhōng*, *Tài Cù*, *Gū Xiǎn*, *Ruí Bīn*, *Yí Zé*, *Wú Yì*.) The reason for this is that the twelve pitches of the chromatic scale are generated by only six of the Marquis Yi bells: six pitches on the main strike-portions; six pitches on the alternate strike-portions. This chromatic scale was played on the bells, as they were originally arranged on the rack, using a regular zigzag pattern.

What we see with the pitch names of the Marquis Yi bells is the evolution of the later, standardized nomenclature described in the *Lǚshì chūnqiū*. For at the time of Marquis Yi (fifth century BC), different states still had different pitch standards. Various pitch-naming systems are alluded to on the Marquis Yi bells, but only two systems — the one used in Zeng (the state in which Marquis Yi lived) and the one used in Chu (Zeng's powerful neighbor) — are adequately described. The Zeng system, described above, was used to name the bell pitches; the Chu system, described below, was used to name the chimestone pitches.

Like the Zeng system, the Chu system named six pitches with six distinct names: *Gū Xiǎn*, *Ping Huáng*, *Wen Wang*, *Xin Zhōng*, *Shou Zhōng,* and *Mu Zhōng* (all but one name differing from those used in Zeng). But the Chu system then goes on to give names for the remaining six pitches. This is done by using the prefix *zhuó* (濁), which is added to each of the above six names. This prefix indicates that the pitch is flattened by one semitone. For example, *zhuó Gū Xiǎn* is the pitch *Gū Xiǎn*, flattened. This detail of the Chu system is confirmed by a set of inscribed pitchpipes from a fourth century BC Chu tomb at Jiangling, Hubei (see map: Figure 34), where not only are the six flattened pitches given the prefix *zhuó*,

49 Bagley, "Prehistory of Chinese Music," 68n19.

the six distinctly named pitches are given the prefix *dìng* (定).[50] *Zhuó* translates as "muddy; turbid; murky," while *dìng* translates as "fixed; definite; calm."

According to V. Mair,[51] the term *dìng* is related to the term *qīng* (清) which translates as "pure; clear; calm." By circa 200 BC, the terms *zhuó* and *qīng* were standard musical terminology:

> the *qīng/zhuó* terminology is attested in Chinese musical terminology early on, where it is widely used [in pre-Qin and Han texts] to refer to the balancing of the tones on a pentatonic scale (*wuyin* 五音), the tuning of pitchpipes ... and as a metaphor for political and cosmological "harmony" or even the selection range in divination. Clear and muddy are, so to speak, "negative polarity items" opening a space from which the ear has to make a selection to produce well-tuned music — and the ruler to produce well-adjusted political measures.[52]

Let's summarize. Sets of bells from before the fifth century BC give no definite proof of a particular scale being used in China: the earliest bell sets suggest a three-note scale or a four-note scale — and, in one case (the Changsha set, circa 1100 BC), the possible knowledge of chromatic semitones. Later, in the Chunqiu period (771–476 BC) there may be evidence for the use of both a pentatonic and a heptatonic scale. Yet it is only with the bells and chimestones of Marquis Yi (433 BC) that there is definite proof of the knowledge of the twelve-tone chromatic scale, of the pentatonic scale, of (possibly) a heptatonic scale (see footnote 49), of transposition of the pentatonic modes, and of the use of both relative and absolute pitch names, varying between states.

Almost all the evidence given above, detailing the tonal system of ancient China, has been derived by examining sets of bronze bells. It must be noted, therefore, that current scholarship[53] increasingly suggests that the ancient Chinese did not discover bronze independently.

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50 Li Chunyi, *Zhongguo shanggu chutu yueqi zonglun* (Beijing, 1996), 377–381.

51 V. H. Mair, personal correspondence.

52 https://languagelog.ldc.upenn.edu/nll/?p=49322 (see comment by W. Behr).

53 https://www.nature.com/articles/srep23304.

For example, a 2017 paper by several researchers proposes that the first meeting between cultures from the West and East occurred around 2000 BC, in the Hexi Corridor. According to this paper, evidence indicates that the Eastern people had contact with the people of the Loess Plateau (near Erlitou: the site of China's earliest bronzes). It is suggested that it was through this exchange that knowledge of the manufacture of bronze and other technologies first entered China:

> Our results show that millet cultivators who used painted potteries from the western Loess Plateau first settled the Hexi Corridor around 4800 BP. Communities who cultivated wheat and barley moved into this region from the west around 4000 BP, bringing with them technologies and materials not seen in central China before, including bronze metallurgy, mud bricks, and mace heads. This was part of the east–west contact which became evident in the Hexi Corridor since the late fifth millennium BP, and continued over the subsequent two millennia, and predated the formation of the overland Silk Road in the Han Dynasty (202 BC–220 AD).[54]

We must consider the possibility, therefore, that all bronze bells found in China were cast at a time when east–west contact had already been established, via the Hexi Corridor. As was previously mentioned, the Hexi Corridor connects the East Asian Heartland with the Tarim Basin, where harps of Mesopotamian design, dating circa 1000 BC, have been found. Further evidence for east–west contact is suggested by glass beads found in the tomb of Marquis Yi. These beads have recently been analyzed by means of X-ray fluorescence spectrometry and are found to have been imported from the west — probably from the eastern Mediterranean and Iran.[55]

We see, therefore, that it is probable that ancient Chinese bronze bells do not pre-date east–west contact. But what about chimestones, which are not related to the manufactured of bronze? Are there earlier sets of chimestones that can shed light on the nature of Chinese music before the earliest

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54 Guanghui Dong, et al., "Prehistoric Trans-continental Cultural Exchange in the Hexi Corridor, Northwest China," *The Holocene*, 28 (4) (2018): 621–628.

55 https://onlinelibrary.wiley.com/doi/epdf/10.1002/jrs.5177

suggested date for east–west contact (circa 2000 BC)? Apparently not, for according to Bagley, “though chime stones go back to the late third millennium BC, Neolithic examples are always found singly; tuned sets do not seem to be any earlier than tuned sets of bells.” [56]

To summarize, there is no concrete evidence for knowledge of bronze manufacture, of stringed instruments, of the *sanfen sunyi* method, of the pentatonic scale, or of the diatonic scale prior to 2000 BC — the proposed date of east–west contact, supported by archaeological discoveries in Xinjang. Let’s look, now, at evidence from ancient Chinese texts that discuss music theory.

56 Bagley, “Prehistory of Chinese Music,” 58.

9. THE ANCIENT CHINESE TONAL SYSTEM: SCALES AND MODES

The earliest description of a scale is given in Chapter 58 of the *Guanzi* (fifth century BC – first century BC), where the pentatonic scale is described as generated by the *sanfen sunyi* method:

> To create the sounds of the five-note scale, first take the primary unit and multiply it by three. Carried out four times, this will amount to a combination of nine times nine [or eighty-one], thereby establishing the pitch of the *Huáng Zhōng* … [the] *gong* note. Adding one-third to make 108 creates the *zhǐ* note. Subtracting one-third results in the appropriate number for producing the *shāng* note. Adding one-third is the means to achieve the *yǔ* note. Subtracting one-third results in the appropriate number for achieving the *jué* note. [57]

> 凡將起五音，先主一而三之，四開以合九九,以是生黃鐘小素之首，以成宮：
> 三分而益之以一，為百有八，為徵；不無有三而去其乘，適足以是生商；
> 有三分而復於其所，以是生羽, 有三分而去其乘以是生角.

57 *Guanzi* (管子), chapter 58, trans. Allyn Rickett, *Guanzi. Political, Economic, and Philosophical Essays from Early China, Volume 2* (Princeton University Press, 1998), 263.

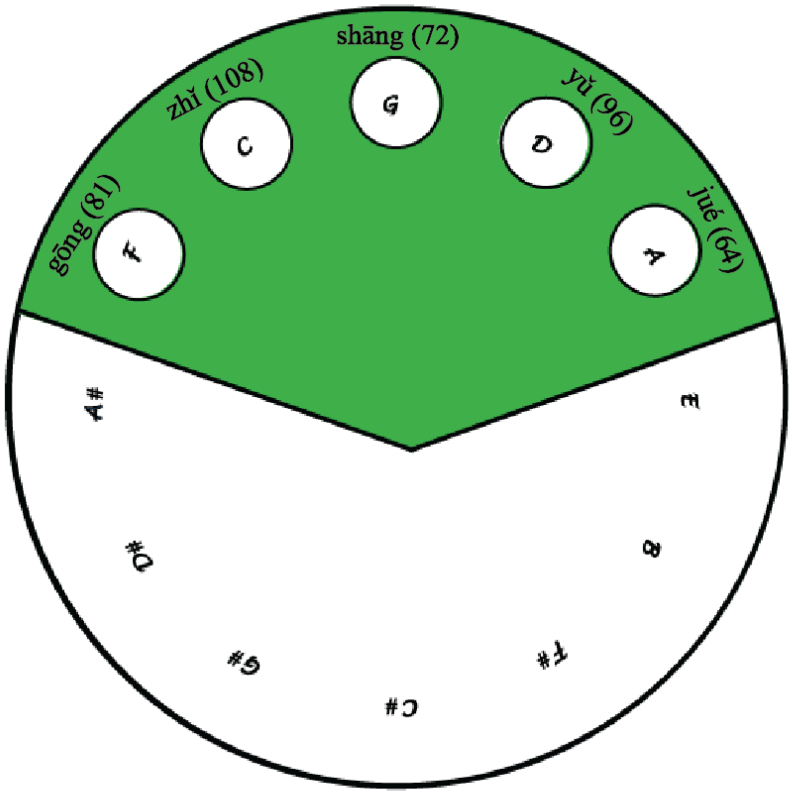

Figure 40. Relationship between the Pentatonic Scale and the Circle of Fifths

The first part of the *Guanzi* text indicates that the note *gōng* is to be given the value of 81 — probably so that the string lengths of all five notes of the scale can be expressed as whole numbers. The text then describes that the note *zhǐ* is derived by up-generation from the note *gōng*. This means that *gōng* cannot be the lowest note of the scale: instead, it is *zhǐ* that is the fundamental. It is for this reason that in Figure 40 the note *zhǐ* is assumed to be C, and the note *gōng*, F — for the note *gōng* must be the *fourth* of *zhǐ*, because when the string length of *gōng* is up-generated (i.e., increased by 1/3 of its value: 81 × 4/3 = 108), the note *zhǐ* is generated.

The next operation described in the text is a down-generation from the note *zhǐ*, to generate the note *shāng* (108 × 2/3 = 72). An up-generation then gives the note *yǔ* (72 × 4/3 = 96). Finally, a down-generation gives the note *jué* (96 × 2/3 = 64).

We see, therefore, that the first five pitches generated by the *sanfen sunyi* method (or up-and-down principle) create a pentatonic scale. Because of the nature of this method, these pitches will be five consecutive notes on the circle of fifths (Figure 40).

Because string length values are proportionate, we can give the fundamental any value and calculate the other notes in the scale accordingly. For example, we can give *zhǐ* a string length of 2 (the value of the fundamental in Table 7, the first twelve lines of which are reproduced as Table 9), causing the notes in the scale to have the values: *zhǐ* = 108 × 2/108 = 2; *yǔ* = 96 × 2/108 = 16/9; *gōng* = 81 × 2/108 = 3/2; *shāng* = 72 × 2/108 = 4/3; *jué* = 64 × 2/108 = 32/27.

As expected, these values (printed in red in Table 9) are embedded within the twelve-tone scale that we generated earlier using the *sanfen sunyi* method. Notice that, because the fundamental, C, is derived from up-generation, F has a string length of 3/2 *, instead of 262144/177147 (the value given for this note in line 6 of Table 7).

Table 9. C *zhǐ* Mode Embedded within the Twelve-tone Chromatic Scale

Line	Notes of C *zhǐ* Mode (in red)	$(2/3)^F \times (2)^D$	Fraction Equivalent
1	C (Fundamental)	$(2/3)^0 \times 2^1$	2
2	C#	$(2/3)^7 \times 2^5$	4096/2187
3	D	$(2/3)^2 \times 2^2$	16/9
4	D#	$(2/3)^9 \times 2^6$	32768/19683
5	E	$(2/3)^4 \times 2^3$	128/81
6	F	$(2/3)^{11} \times 2^7$	3/2 *
7	F#	$(2/3)^6 \times 2^4$	1024/729
8	G	$(2/3)^1 \times 2^1$	4/3
9	G#	$(2/3)^8 \times 2^5$	8192/6561
10	A	$(2/3)^3 \times 2^2$	32/27
11	A#	$(2/3)^{10} \times 2^6$	65536/59049
12	B	$(2/3)^5 \times 2^3$	256/243

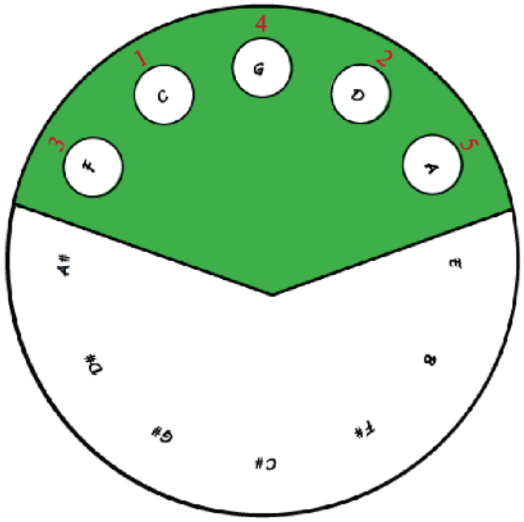

Figure 41. C *zhǐ* Mode (with Up-Generated Fundamental): 3, 1, 4, 2, 5

The red numbers in Figure 41 (3, 1, 4, 2, 5) select the five pitches printed in red in Table 9 from the circle of fifths, ordering them from lowest to highest, thereby generating an ascending pentatonic scale in the mode C *zhǐ*: (C) *zhǐ*: 1st note; (D) *yǔ*: 2nd note; (F) *gōng*: 3rd note; (G) *shāng*: 4th note; (A) *jué*: 5th note. Inversions of the sequence 3, 1, 4, 2, 5 generate the pentatonic modes.

For example, if we let *gōng* be the fundamental, having the pitch C, it will be the first of five consecutive fifths (Figure 42). Now, a down-generation gives G: 2 × 2/3 = 4/3. Next, an up-generation gives D: 4/3 × 4/3 = 16/9. Then, a down-generation gives A: 16/9 × 2/3 = 32/27. Finally, an up-generation gives E: 32/27 × 4/3 = 128/81. Ordering the string lengths from longest to shortest gives the mode C *gōng*: C 1st note; D: 2nd note; E: 3rd note; G: 4th note; A: 5th note.

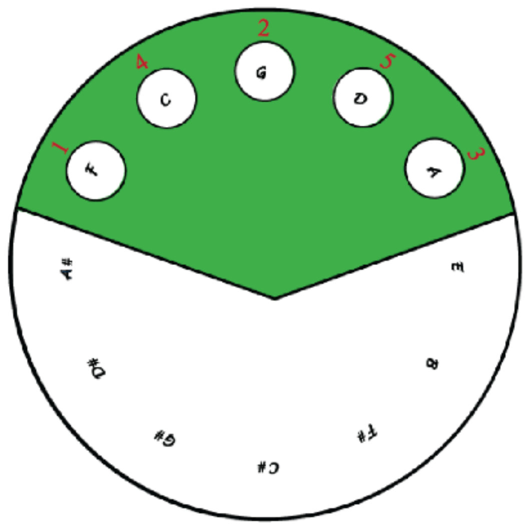

Figure 42. C *gōng* Mode (without Up-Generated Fundamental): 1, 4, 2, 5, 3

Table 10. C *gōng* Mode Embedded within the Twelve-tone Chromatic Scale

Line	Notes of C Gōng Mode (in red)	$(2/3)^F \times (2)^D$	Fraction Equivalent
1	C (Fundamental)	$(2/3)^0 \times 2^1$	2
2	C#	$(2/3)^7 \times 2^5$	4096/2187
3	D	$(2/3)^2 \times 2^2$	16/9
4	D#	$(2/3)^9 \times 2^6$	32768/19683
5	E	$(2/3)^4 \times 2^3$	128/81
6	F	$(2/3)^{11} \times 2^7$	262144/177147
7	F#	$(2/3)^6 \times 2^4$	1024/729
8	G	$(2/3)^1 \times 2^1$	4/3
9	G#	$(2/3)^8 \times 2^5$	8192/6561
10	A	$(2/3)^3 \times 2^2$	32/27
11	A#	$(2/3)^{10} \times 2^6$	65536/59049
12	B	$(2/3)^5 \times 2^3$	256/243

Table 10 duplicates Table 9, but highlights the pitches of this C *gōng* mode. As before, this mode is embedded in the twelve-tone scale generated by the up-and-down principle.

Notice the sequence of numbers in Figure 42 that selects the five pitches printed in red in Table 10 from the circle of fifths: 1, 4, 2, 5, 3. This sequence generates an ascending pentatonic scale in the mode C *gōng*: It is an inversion of the sequence in Figure 41. It is, in fact, embedded in the instructions in the *Lǚshì chūnqiū* that describe the *sanfen sunyi* method (column 3, Table 2).

Because there are five possible inversions of this sequence and twelve possible rotations (i.e., when the green shape in Figure 42 is rotated, hole 1 can indicate any of twelve possible notes), there are 60 pentatonic modes. Evidence that the Chinese conceptualized the pentatonic modes in this way (i.e., as a mathematical set) is found in the *Huainanzi* (120 BC): "One pitch generates a set of five notes. Twelve pitches for a set of sixty notes. [一律而生五音，十二律而為六十音.]" [58]

Let's look, now, at the ancient Chinese heptatonic scale, applying the *sanfen sunyi* method to generate the scale played by the Biao bells, described earlier: *do*, *re*, *mi*, *fa*, *sol*, *la*, *ti* (or C, D, E, F, G, A, B). First, we must recognize that, as with the pentatonic scale described in the *Guanzi* (Figure 41), the fundamnetal of this scale is up-generated — because the scale contains an F, rather than an F#. We will give this F the value of 729 because this is the smallest number that will allow the string lengths of all the notes in the scale to be expressed as whole numbers.

We begin with F = 729. An up-generation then gives C = 972 (because 729 × 4/3 = 972). A down-generation then gives G = 648 (because 972 × 2/3 = 648). An up-generation then gives D = 864 (because 648 × 4/3 = 864). A down-generation then gives A = 576 (because 864 × 2/3 = 576). An up-generation then gives E = 768 (because 576 × 4/3 = 768). Last, a down-generation gives B = 512 (because 768 × 2/3 = 512). As before, because string length values are proportionate, we can give C any value and adjust the other pitches accordingly. For example, we can give C the value of 1. The remaining values will then be: D = 864/972 = 8/9; E = 768/972 = 64/81; F = 729/972 = 3/4; G = 648/972 = 2/3; A = 576/972 = 16/27; B = 512/972 = 128/243. Note that these are the values given by Needham and Robinson in Figure 22 for the Greek (Pythagorean) scale.

58 *Huainanzi* (淮南子), Chapter Three "Celestial Patterns" (天文训第三), trans. C. Y. Chen, in *Early Chinese Work in Natural Science* (Hong Kong University Press, 1996), 108.

In Table 11 the notes of this scale — known today, in the West, as C Ionian mode (the ancient Greeks called it the Lydian mode) — are printed in red. As was done in Tables 7, 9 and 10, the fundamental in Table 11 is given a value of 2. The values for other notes of the scale then become 16/9, 128/81, 3/2, 4/3, 32/27 and 256/243. As before, the notes of this scale, printed in red, are embedded in the twelve-tone chromatic scale generated by the *sanfen sunyi* method.

Table 11. C Ionian Mode (Ancient Greek Lydian Mode) Embedded within the Twelve-tone Scale

Line	Notes of C Ionian Mode (in red)	$(2/3)^F \times (2)^D$	Fraction Equivalent
1	C (Fundamental)	$(2/3)^0 \times 2^1$	2
2	C#	$(2/3)^7 \times 2^5$	4096/2187
3	D	$(2/3)^2 \times 2^2$	16/9
4	D#	$(2/3)^9 \times 2^6$	32768/19683
5	E	$(2/3)^4 \times 2^3$	128/81
6	F	$(2/3)^{11} \times 2^7$	3/2
7	F#	$(2/3)^6 \times 2^4$	1024/729
8	G	$(2/3)^1 \times 2^1$	4/3
9	G#	$(2/3)^8 \times 2^5$	8192/6561
10	A	$(2/3)^3 \times 2^2$	32/27
11	A#	$(2/3)^{10} \times 2^6$	65536/59049
12	B	$(2/3)^5 \times 2^3$	256/243

As shown in Figure 43, the nature of the up-and-down principle dictates that the notes in this seven-note scale are consecutive notes on the circle of fifths. The red numbers in Figure 43 select the pitches printed in red in Table 11 from the circle of fifths, giving an ascending diatonic scale: C (2) 1st note; D (16/9): 2nd note; E (128/81): 3rd note; F (3/2): 4th note; G (4/3): 5th note; A 32/27): 6th note; B (256/243): 7th note. Notice the sequence of numbers generated: 4,1,5,2,6,3,7.

Earlier, we constructed the pentatonic scale in two ways: from a fundamental derived by up-generation (Figure 41), and from a fundamental *not* derived by up-generation (Figure 42). The same options are possible with the diatonic scale: Figure 43 shows a scale with a fundamental that is up-generated; Figure 44 shows a scale with a fundamental that is *not* up-generated.

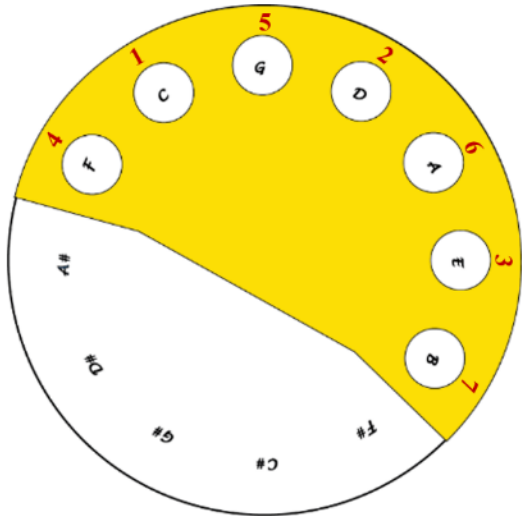

Figure 43. Chinese Heptatonic Scale with Up-Generated Fundamental (also C Ionian Mode): 4,1,5,2,6,3,7

To construct the scale in Figure 44, a down-generation is applied to C = 2 to give G: 2 × 2/3 = 4/3. An up-generation then gives D: 4/3 × 4/3 = 16/9. Next, a down-generation gives A: 16/9 × 2/3 = 32/27. Then, an up-generation gives E: 32/27 × 4/3 = 128/81. Next, a down-generation gives B: 128/81 × 2/3 = 256/243. Finally, an up-generation gives F#: 256/243 × 4/3 = 1024/729. This scale, printed in red in Table 12, is known, in the West, as the C Lydian mode (the ancient Greek name was Hypolydian). Notice the red numbers in Figure 44: 1,5,2,6,3,7,4. These numbers select the pitches printed in red in Table 12 from the circle of fifths, arranging them in ascending order. Recall that we saw this sequence earlier, embedded in the instructions in the *Lǚshì chūnqiū*.

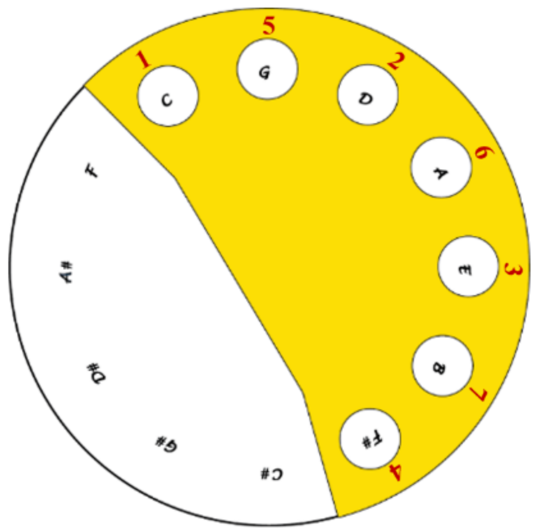

Figure 44. Chinese Heptatonic Scale without Up-Generated Fundamental (also C Lydian Mode): 1,5,2,6,3,7,4

Table 12: C Lydian Mode (Ancient Greek Hypolydian mode) Embedded within the Twelve-tone Scale

Line	Notes of C Lydian Mode (in red)	$(2/3)^F \times (2)^D$	Fraction Equivalent
1	C (Fundamental)	$(2/3)^0 \times 2^1$	2
2	C#	$(2/3)^7 \times 2^5$	4096/2187
3	D	$(2/3)^2 \times 2^2$	16/9
4	D#	$(2/3)^9 \times 2^6$	32768/19683
5	E	$(2/3)^4 \times 2^3$	128/81
6	F	$(2/3)^{11} \times 2^7$	262144/177147
7	F#	$(2/3)^6 \times 2^4$	1024/729
8	G	$(2/3)^1 \times 2^1$	4/3
9	G#	$(2/3)^8 \times 2^5$	8192/6561
10	A	$(2/3)^3 \times 2^2$	32/27
11	A#	$(2/3)^{10} \times 2^6$	65536/59049
12	B	$(2/3)^5 \times 2^3$	256/243

As mentioned before, there is evidence for early use of a heptatonic scale with up-generated fundamental: this is the scale played by both the "Biao" bells and the nine-bell set from Henan. Is there also evidence for the use of a heptatonic scale with a fundamental that is not up-generated?

As cited earlier (see footnote 38), the *niu*-bells (771 BC - 476 BC) can produce "heptatonic scales with major sevenths and raised fourths." The scale shown in Figure 44, created when the fundamental is not up-generated, is such a scale. Wei Zhao (204–273 AD) believed that this scale is alluded to in the *Guo Yu* (fifth century BC), when mention is made of "seven pitches." His commentary on the relevant *Guo Yu* passage (which is cited in footnotes 188 and 189) is as follows:

> *Discourses of the States*, "Discourses of Zhou", C: "What are the seven [pitch] standards?" Note by Wei Zhao: The seven [pitch] standards are tuning devices. *Huáng Zhōng* is used for *gōng*. *Tài Cù* is used for *shāng*. *Gū Xiǎn* is used for *jué*. *Lín Zhōng* is used for *zhǐ*. *Nán Lǚ* is used for *yǔ*. *Yìng Zhōng* is used for *biangong* and *Ruí Bīn* is used for *bianzhi*." [59]
>
> 七律為何? 七律為音器，用黃鐘為宮，太簇為商，姑洗為角，林鍾為徵，南呂為羽，應鐘為變宮，蕤賓為變徵也.

If the *Huáng Zhōng* is assumed to be C, the pitches given by Wei Zhao are: *Huáng Zhōng* (C); *Tài Cù* (D); *Gū Xiǎn* (E); *Lín Zhōng* (G); *Nán Lǚ* (A); *Yìng Zhōng* (B); and *Ruí Bīn* (F#). [Recall that the prefix *bian* indicates flattening by one semitone. Therefore, if the *Huáng Zhōng* (or *gōng*) is C, *biangōng* is B. Similarly, if *Lín Zhōng* (or *zhǐ*) is G, *bianzhǐ* is F#.] Arranged in chromatic order, these pitches generate the scale shown in Figure 44: C, D, E, F#, G, A, B.

To summarize, the *sanfen sunyi* method allows two options for the fundamental: 1) an up-generated fundamental (Figure 41 and Figure 43); 2) a fundamental that is not up-generated (Figure 42 and Figure 44). With regards to the diatonic scale, this is why we have only seen the use, so far, of two inversions of the seven-number sequence: 4,1,5,2,6,3,7 and 1,5,2,6,3,7,4. Once the scale is derived,

59 Wei Zhao, *Annotations to the Guo Yu* (国语注), trans. Victor H. Mair, personal correspondence.

however, the other inversions of this sequence can be used to generate the other diatonic modes. As we'll see, these modes are the seven tunings used by the Mesopotamians.

It is important to note that we have now seen *two applications* of the sequence 4,1,5,2,6,3,7. For, earlier, we saw that it operates as part of a "computer" that calculates the string lengths of the notes in a *twelve-tone* scale. Yet now we see that it selects the notes in the *seven-note* diatonic modes from the circle of fifths. We'll discuss the relationship between these two applications of sequence shortly but first, let's summarize our discussion of the history of music in China.

As previously mentioned, twenty-three angular harps of Mesopotamian design have been found in Xinjiang — some dating to as early as 1000 BC. Can we hypothesize a connection between these harps and the musical scales suggested by the sets of bells found in the East Asian Heartland?

As mentioned earlier, knowledge of bronze metallurgy was probably transmitted to the heartland of China circa 2000 BC — almost a millennium before the date of the earliest Xinjiang harps. Therefore, bronze signalling bells were probably already in use in China before the first angular harps began to appear from the west, and were perhaps used, in their first musical applications, to accompany the three- and four- note scales previously described (see footnote 37) — scales that formed the basis of musical compositions that were played, it is suggested here, by flutes and individual chimestones, but not by stringed instruments.

As we saw earlier (see footnote 10), Lawergren suggests that the Xinjiang harps evolved into the ancient *qin* during the early first millennium BC and that this *qin*-like instrument then entered the East Asian Heartland via the Hexi Corridor. It is proposed here that knowledge of how to tune these instruments — by consecutive fifths, in keeping with the Mesopotamian tonal system — and the scales generated by this method of tuning, were transmitted with them. It was then, it is suggested, that pentatonic and heptatonic scales began to be used in China, for as we have seen, evidence of their use first appears with the bell sets of the Chunqiu period (771 BC – 476 BC).

It is also proposed here that knowledge of the mathematics inherent in the Mesopotamian tonal system (i.e., the mathematics that underlie the *sanfen sunyi* method) was not immediately transmitted with the harps. For, as is true today, a musician need not know the mathematics that underlie the construction of scales in order to play and tune an instrument. Instead, it is suggested that this transmission occurred around the sixth century BC — early enough that when the Marquis Yi bells were made the generation of the *shí'èr lǜ* by the *sanfen sunyi* method was fully understood.

Let's review what we've learned about the *sanfen sunyi* method. Twelve fifths span seven octaves — the difference being the Pythagorean comma. As we have seen, the instructions in the *Lǚshì chūnqiū* imply knowledge of this comma. These instructions also suggest knowledge of the true octave (1/2), for when a down-generation will cause a fifth to be projected outside of the octave under construction, an up-generation is used instead, causing the string length of the fifth in question to be doubled, thereby generating its lower octave.

Tablet UET VII 74, which we will look at shortly, suggests that the Mesopotamians also had knowledge of these three things: the true octave, the fifth, and the Pythagorean comma. There is only one further interval that we need to identify, in order to reconstruct the Mesopotamian tonal system: the interval that, today, we call a "tritone." We have seen that the diatonic scale is constructed from seven consecutive fifths. As shown by the pink arrows in Figure 45, the interval between six of these fifths is seven semitones. The interval between the first and last fifth (for example: B to F), however, is only six semitones (or three tones). This is the interval of the tritone.

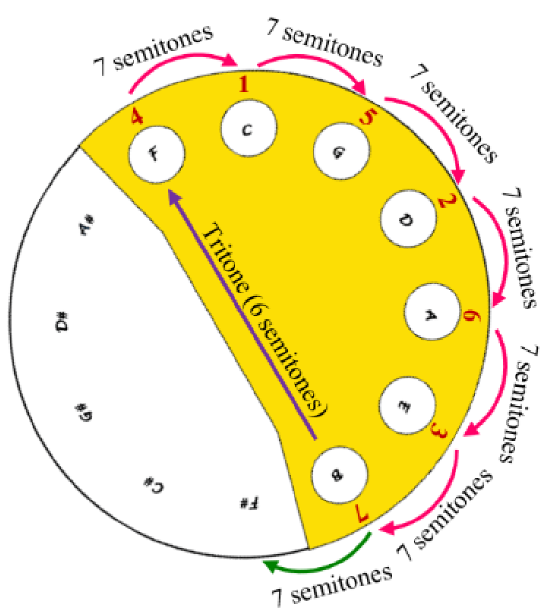

Figure 45. 7 Fifths = 49 semitones; 6 Fifths + 1 Tritone = 48 semitones

10. THE MESOPOTAMIAN TONAL SYSTEM: FOUR CUNEIFORM TABLETS

Four cuneiform tablets, translated during the last sixty years, allow us to reconstruct the Mesopotamian tonal system.

Tablet UET VII 126 (Figure 46) dates to approximately 600 BC. A fragment of another tablet that dates from a millennium earlier duplicates some of the information on UET VII 126, indicating that the system was in use for over a thousand years.

Figure 46. UET VII 126, circa 600 BC (Iraq Museum, Iraq)

Table 13. String Names as Given on UET VII 126

String no.	String Name
1	foremost or front string
2	next or second string
3	third, thin string
4	fourth, small/ string created by Ea
5	fifth string
6	fourth behind string
7	third behind string
8	second behind string
9	behind string

UET VII 126 lists the nine strings of the *sammû*, the Mesopotamian lyre. The strings were identified by counting inwards from the front and back of the instrument towards the middle, and their names reflect this. A translation of the relevant text from UET VII 126 is given in Table 13. The names of the first seven strings in Table 13 are printed in red, because it is these names that are written on the heptagram on tablet CBS 1766 (Figure 16), as described earlier.

CBS 10996 (Figure 47) dates to approximately 700 BC. This tablet gives the names of fourteen intervals, heard by playing pairs of the seven strings printed in red in Table 13. These intervals are listed in Table 14.

Figure 47. CBS 10996, circa 700 BC (Univ. of Pennsylvania Museum)

Table 14. Intervals as given on CBS 10996

Line	String No.	Interval Name	Translation
1	1 → 5	*nīš tuḫrim*	raising of the counterpart
2	7 → 5	*serum*	song?
3	2 → 6	*išartum*	straight
4	1 → 6	*salsatum*	third
5	3 → 7	*embūbum*	reed-pipe
6	2 → 7	*rebutum*	fourth
7	4 → 1	*nīd qablim*	casting down the middle
8	1 → 3	*isqum*	lot/portion
9	5 → 2	*qablītum*	middle
10	2 → 4	*titur qablitim*	bridge of the middle
11	6 → 3	*kitmum*	covering / closing
12	3 → 5	*titur isartim*	bridge of the isartum
13	7 → 4	*pītum*	opening
14	4 → 6	*s/zerdum*	?

On CBS 10996, an interval is indicated by the numbers of the two strings that bound it, and also by a name. For example, the interval heard between strings 5 and 2 is written *5 → 2* and is called *qablītum* (line 9, Table 14). Notice the sequence 1,5,2,6,3,7,4 in column 2 of Table 14.

The "tuning text" mentioned earlier (UET VII 74, Figure 13) is the last tablet in the group. It allows us to use the information provided by CBS 1766, UET VII 126, and CBS 10996 to arrive at an overall picture of the system. UET VII 74 describes how to tune the *sammû* to seven different tunings. The names of these tunings are the same names that are given to seven of the intervals listed on CBS 10996 (the names printed in red in Table 14). The text of UET VII 74 is as follows:

If the *sammû* is *išartum* and *qablītum* is not clear, tighten string 5 and *qablītum* will be clear.

If ″ ″ is *qablītum* and *nīš tuḫrim* is not clear, tighten string 1 & 8 and *nīš tuḫrim* will be clear.

If ″ ″ is *nīš tuḫrim* and *nīd qablim* is not clear, tighten string 4 and *nīd qablim* will be clear.

If ″ ″ is *nīd qablim* and *pītum* is not clear, tighten string 7 and *pītum* will be clear.

If ″ ″ is *pītum* and *embūbum* is not clear, tighten string 3 and *embūbum* will be clear.

If ″ ″ is *embūbum* and *kitmum* is not clear, tighten string 6 and *kitmum* will be clear.

If ″ ″ is *kitmum* and *išartum* is not clear, tighten string 2 & 9 and *išartum* will be clear.

If ″ ″ is *išartum* and *qablītum* is not clear, loosen string 2 & 9 and the *sammû* will be *kitmum*.

If ″ ″ is *kitmum* and *išartum* is not clear, loosen string 6 and ″ ″ will be *embūbum*.

If ″ ″ is *embūbum* and *kitmum* is not clear, loosen string 3 and ″ ″ will be *pītum*.

If ″ ″ is *pītum* and *embūbum* is not clear, loosen string 7 and ″ ″ will be *nīd qablim*.

If ″ ″ is *nīd qablim* and *pītum* is not clear, loosen string 4 and ″ ″ will be *nīš tuḫrim*.

If ″ ″ is *nīš tuḫrim* and *nīd qablim* is not clear, loosen string 1 & 8 and ″ ″ will be *qablītum*.

If ″ ″ is *qablītum* and *nīš tuḫrim* is not clear, loosen string 5 and ″ ″ will be *išartum*.

In the text, the *sammû* "is" a particular name — *išartum*, for example. This is understood to mean that the *sammû* "is in the tuning *isartum*." Furthermore, in the *išartum* tuning, the text states that *qablītum* is "not clear." In this case, *qablītum* is understood to refer to the interval *qablītum* (not the tuning *qablītum*): the interval between strings 5 and 2 (see line 9, Table 14).

Notice that, in every case, the name of the interval that is "not clear" is the same name that is given to the subsequent tuning — the tuning that results from making this interval "clear." For example, the *išartum* tuning, in which *qablītum* is "not clear," is followed by the tuning *qablītum*.

The text describes the sequential tuning of a series of strings: the 5th string is tuned, then the 1st (and 8th), then the 4th, 7th, 3rd, 6th, and 2nd (and 9th). Because the text indicates that strings 1 and

8 are tuned at the same time, and also strings 2 and 9, the consensus among scholars is that string 8 is the perfect octave of string 1, and string 9 is the perfect octave of string 2. Consequently, there are only seven distinct pitches: those played by strings 1 to 7. It becomes apparent, therefore, why CBS 1766 makes reference to only the first seven string names, and also why CBS 10996 only lists intervals bounded by these same seven strings: the Mesopotamian system is heptatonic.

When we remove the duplicate strings 8 and 9 from the sequence that gives the order of strings tuned, we get 5,1,4,7,3,6,2. This is an inversion of 4,1,5,2,6,3,7 but in reverse order. Listing the second set of seven strings tuned, we get 2,6,3,7,4,1,5. This is also an inversion of 4,1,5,2,6,3,7 but, in this case, not reversed. In fact, the two sequences given in the text are mirror images: 5,1,4,7,3,6,2 → 2,6,3,7,4,1,5. This realization led scholars to deduce that the first seven "sentences" of the text describe a process that is then reversed in the second group of seven sentences. These two groups have come to be called "chapters." On UET VII 74, Chapter 1 is headed by the word "*nu-su-h*[*u-um*]," meaning "to tighten." It is the general consensus, therefore, that Chapter 1 describes the tightening of each consecutive string, while Chapter 2 describes their loosening.

It is important to note that UET VII 74 does not explain the mathematics behind the construction of the tunings it describes. Instead, the instructions on the tablet assume that the *sammû* is already tuned, and simply describe how to *re-tune* the *sammû* in order to move from one tuning (mode) to another. Chapter 1 cycles through seven tunings; Chapter 2 reverses the process.

The fact that none of the tablets that relate to the Mesopotamian tonal system describe, specifically, that the system was based on mathematics, has led to debate among music archaeologists. Some believe that no mathematical basis can be deduced while others argue that the Mesopotamians, who display (on other, unrelated tablets) an in-depth understanding of mathematics, would surely have explored the relationship between string length and pitch.

What we will see, now, is that the heptagram we derived earlier using the *sanfen sunyi* method illustrates, *exactly*, the re-tuning process described on UET VII 74, suggesting that the Mesopotamian tonal system is, in fact, based on an awareness of the mathematics of music.

11. THE HEPTAGRAM ON CBS 1766: REGULAR OR IRREGULAR?

In 2007, CBS 1766 was first understood to relate to music — for from the time of its discovery in the 1890s until 2007, the tablet was thought to be "an astrological scheme to relate the seven ancient planets to the seven days of the week." [60] Then, in 2007, Assyriologists Waerzeggers and Siebes recognized the writing on the points of the heptagram to be the names of seven of the strings of the *sammû*. Consequently, they proposed that:

> the seven-pointed star of CBS 1766 [is] a visual tuning chart supplementing the numerical and verbal instructions contained in theoretical musical texts such as CBS 10996. [61]

Yet, to date, no model of a "visual tuning chart" that correctly illustrates the tuning instructions on UET VII 74 has been conceived. The problem lies in the fact that although scholars agree that the tuning instructions on UET VII 74 imply the generation of twelve distinct pitches (we will see why, shortly), they assume that the heptagram on CBS 1766 is meant to be regular (i.e., that it is meant to have seven equidistant points). However, even when rotated, a regular heptagram can only indicate seven positions on a circumscribed circle (Figure 48). Consequently, when arranged circularly, the twelve pitches generated by the instructions on UET VII 74 cannot be indicated by the points of the heptagram, if the heptagram is assumed to be regular.

60 A. Jeremias, *Handbuch der alterorientalischen Geisteskultur*, 2nd ed. (Berlin and Leipzig, 1929), 197–199.

61 C. Waerzeggers and R. Siebes, "An Alternative Interpretation of the Seven-Pointed Star on CBS 1766," *NABU* (Nouvelles Assyriologiques Brèves et Utilitaires) /2 (2007): 43–45.

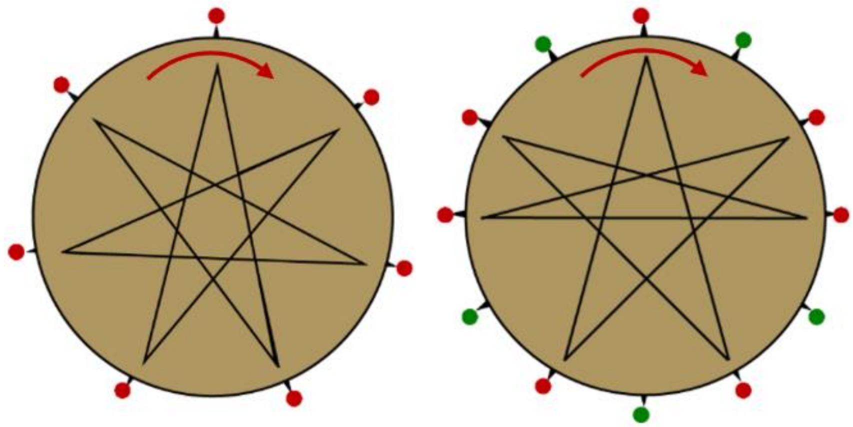

Figure 48. CBS 1766: Regular Heptagram vs. Irregular Heptagram

What we will see now, however, is that the irregular heptagram that we derived earlier is able, when rotated on a circle, to indicate twelve different positions — and therefore twelve pitches — and that this model exactly illustrates the tuning instructions on UET VII 74.

12. THE MESOPOTAMIAN TONAL SYSTEM: THE SEVEN DIATONIC MODES

UET VII 74 was first published in 1968, by O. Gurney, who collaborated with Oxford musicologist D. Wulstan to interpret the meaning of the instructions on the tablet (see translation, given earlier). M. L. West gives the following synopsis of their conclusions:

> As all the tunings are heptatonic (the octave counterpart of 1 or 2 being reached in seven steps), it is fairly clear that these primary intervals are all fifths or fourths. It is true that the interval 2-6, say, will not be the same in all tunings. But in the "tuning text" this variation is expressed by saying that the *išartum* interval is "pure" or "not pure." As Wulstan saw, this must mean "concordant/discordant," that is, "pure fifth/tritone." So *išartum* properly means "strings 2 and 6 tuned to a fifth" — otherwise it is not a pure *išartum* — and similarly with the rest. [62]

To understand West's description, let's return to our earlier discussion of the diatonic scale. As we saw, the notes of the diatonic scale are seven consecutive fifths. Consequently, with respect to the Ionian mode (or ancient Greek Lydian mode) (Figure 45), the interval between notes 4 and 1 is a fifth. So is the interval between notes 1 and 5; notes 5 and 2; notes 2 and 6; notes 6 and 3; and notes 3 and 7. However, the interval between notes 7 and 4 is a tritone.

As explained by West, Gurney and Wulstan identified all seven intervals referred to on UET VII 74 as being — usually — fifths. Yet seven fifths, each spanning seven semitones, together span 49 semitones. (This is shown in Figure 45 by the six pink arrows and one green arrow which, together, span 49 semitones: from F to F#.) In order for the seven intervals to span an octave, therefore, one of these seven fifths must be reduced in size to a tritone, which spans six semitones. (Because six fifths [42 semitones] + a tritone [6 semitones] = 48 semitones, and since 48 is a multiple of 12, an octave of the fundamental is reached. Note: the up-and-down principle ensures that all the notes will lie within the

62 West, "Babylonian Musical Notation," 163.

compass of an octave.] It is for this reason that Gurney and Wulstan interpret the "not pure" or "not clear" interval described on UET VII 74 to be a tritone.

A fifth and a tritone differ by the span of one semitone. Therefore, the act of making an interval "clear," described on UET VII 74, is accomplished by raising one of the notes that bound the interval of the tritone by one semitone. As six of the seven modes contain every interval in its "clear" state, we can identify this state as the norm. It is interesting, therefore, that the ancient Chinese terms *qīng* ("pure; clear; calm") and *zhuó* ("muddy; turbid; murky"), described earlier (see fn 50) "refer to normal vs. flatted musical pitch."[63] Is the use of such closely related adjectives to describe similar musical operations further evidence for west to east transmission?

Figure 49 shows the irregular heptagram derived earlier, overlaid on a circle of twelve notes, but, in this case, the notes are ordered chromatically — as they are listed in the last four lines of the passage from the *Lǚshì chūnqiū*. Notice that the points of the heptagram select the same notes that are indicated by holes 1 to 7 of the yellow overlay in Figure 45. Consequently, we should expect the same interval relationship that we saw in Figure 45 to be found between the notes joined by the diagonals of the heptagram in Figure 49. And this is, in fact, the case:

63 https://languagelog.ldc.upenn.edu/nll/?p=49322 (see comment by D. Prager Branner).

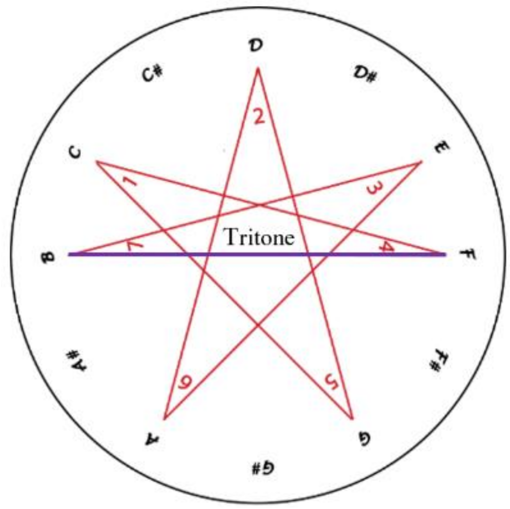

Figure 49. The Irregular Heptagram derived earlier, on a Circle of Twelve Chromatic Notes

- Point 4 indicates F. Point 1 indicates C. So diagonal 4–1 gives the interval of a fifth.
- Point 1 indicates C. Point 5 indicates G. So diagonal 1–5 gives the interval of a fifth.
- Point 5 indicates G. Point 2 indicates D. So diagonal 5–2 gives the interval of a fifth.
- Point 2 indicates D. Point 6 indicates A. So diagonal 2–6 gives the interval of a fifth.
- Point 6 indicates A. Point 3 indicates E. So diagonal 6–3 gives the interval of a fifth.
- Point 3 indicates E. Point 7 indicates B. So diagonal 3–7 gives the interval of a fifth.
- Point 7 indicates B. Point 4 indicates F. So diagonal 7–4 gives the interval of a tritone.

We see, therefore, that when placed on a chromatic circle of twelve notes, the diagonals of the irregular heptagram derived earlier indicate six fifths and one tritone — the exact intervals that Gurney and Wulstan interpret as being used to construct each of the tunings on UET VII 74. But, as previously mentioned, the unresolved question is whether these intervals were calculated mathematically. West, expanding on the work of Gurney and Wulstan, suggests the following:

> The temperament of all these tunings, as they were regulated by perfect fifths and fourths, would have been the so-called Pythagorean, with tones of 204 cents (the difference between a perfect fifth, 702 cents, and a perfect fourth, 498 cents).... An

> "impure" interval arose in each tuning…. We have described it as a tritone, and it was in some cases indeed strictly equal to three tones, 612 cents, but in others … to two tones plus two minor semitones, 588 cents.[64]

The values given by West[65] match those of the ancient Chinese heptatonic scale described in Table 11. For the interval of the fifth described by West has a value of 702 cents, which is the equivalent of dividing line 1 of Table 11 by line 8 (2 ÷ 4/3 = 3/2). Moreover, the interval of the fourth described by West has a value of 498 cents, which is the equivalent of dividing line 1 of Table 11 by line 6 (2 ÷ 3/2 = 4/3). Furthermore, West's tritone interval has a value of 612 cents, which is the equivalent of dividing line 1 of Table 11 by line 7 (2 ÷ 1024/729 = 729/512 = 1.424).[66] Joran Friberg, an expert in Mesopotamian mathematics, also proposes that the pitches described by UET VII 74 were generated using the ratios of the perfect fifth and perfect fourth.[67]

Before we look at how our irregular heptagram, superimposed upon a chromatic circle of twelve notes, can be used to illustrate the instructions on UET VII 74, one final point must be mentioned: the Mesopotamians, like the Greeks who inherited much of their musical system, counted the notes in their scales from the top, downward.

Originally, Gurney and Wulstan assumed that the notes in the tunings described on UET VII 74 were numbered in an ascending direction, as is the tradition with scales in the West, today. Then, a more thorough reading of the tablet [68] showed that in Chapter 1, the re-tuned strings are "tightened," while in Chapter 2, the re-tuned strings are "loosened." It was then realized that, in order for a sequential tuning of fifths to be accomplished, the order of strings tightened in Chapter 1 requires that the notes in the

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64 West, "Babylonian Musical Notation," 164.

65 The cent is a logarithmic unit of measure used for musical intervals. Cents to ratio calculators are available online.

66 Note: 588 cents is a tritone, but measured in the opposite direction. For example, in Table 12, it is the equivalent of dividing line 7 by the octave of the fundamental, which has a value of 1: (1024/729 ÷ 1= 1.405 = 588 cents).

67 Joran Friberg, "Seven-Sided Star Figures and Tuning Algorithms in Mesopotamian, Greek, and Islamic Texts," *Archiv für Orientforschung* 52 (2013): 149–150.

68 Theo. J. H. Krispijn, "Beiträge zur altorienalischen Musikforschung I," *Akkadica* 70 (1990): 1–27.

scales be numbered from the top, down. This is confirmed by Chapter 2, where the order of strings loosened requires the use of the same descending numbering.

To quote M. L. West:

> That string 1 sounded the highest note … has now been proved correct by a new reading of the "tuning text" that makes it clear which re-tunings were effected by tightening strings and which by loosening.[69]

As for which end of the lyre the strings were numbered from, "the 'bovine' lyre had a very obvious front: the animal head, which faced away from the player." [70] The consensus, therefore, is that string 1 — the "foremost or front string" referred to on tablet UET VII 126 (see Table 13) — was closest to the animal head, and played the highest note. The remaining strings were numbered, consecutively, towards the rear of the animal (Figure 50) and generated a descending scale.

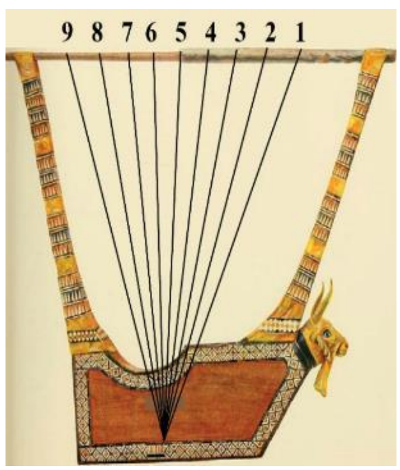

Figure 50. The Numbering of the Strings of the *sammû*

69 West, "Babylonian Musical Notation," 165.

70 West, "Babylonian Musical Notation," 166.

The descending numbering of the Mesopotamian scales is the reason why, in Tables 7, 9, 11, and 12, the fundamental was given a string length of 2. For this format allows us to give the highest note in each Mesopotamian mode, sounded by string 1, a value of 1, and each subsequent value in the descending scale a greater value, until the lower octave, with a value of 2, is reached.

Figure 51 duplicates Figure 49, but adds green numbers, counterclockwise outside the circle. These numbers identify the notes (each note is played by one string) in each mode, in descending order. Notice that the green numbers are in the opposite direction to the numbers on the heptagram. As we'll see, the opposing order of these two sets of numbers will prove useful: they will combine to create a mnemonic device that summarizes the instructions on UET VII 74.

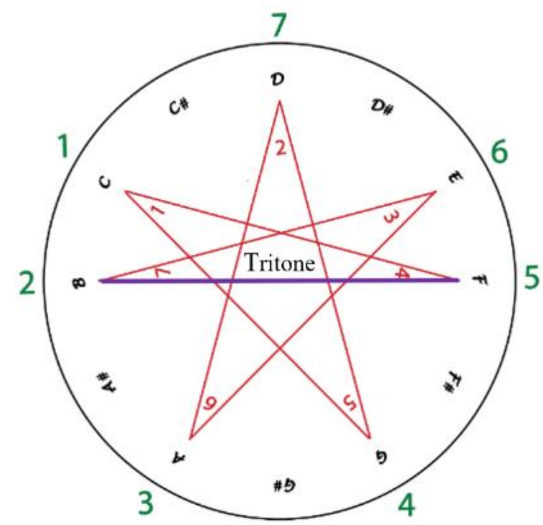

Figure 51. The *išartum* Tuning: C, B, A, G, F, E, D

The first tuning described in Chapter 1 of UET VII 74 is *išartum*:

If the *sammû* is *išartum* and *qablītum* is not clear, tighten string 5 and *qablītum* will be clear.

Let's hypothesize that the *išartum* tuning is shown in Figure 51. Why? Because the "not clear" interval in *išartum* is *qablītum* — between strings 5 and 2 — and in Figure 51 the "tritone line" does, in fact, lie between the notes numbered 5 and 2 (in green).

Table 15. The *išartum* Tuning

Line	Note #	Name	$(2/3)^F \times (2)^D$	Fraction Equivalent
1	9	B	$(2/3)^5 \times 2^4$	512/243
2	8	C	$(2/3)^0 \times 2^1$	2
3		C#	$(2/3)^7 \times 2^5$	4096/2187
4	7	D	$(2/3)^2 \times 2^2$	16/9
5		D#	$(2/3)^9 \times 2^6$	32768/19683
6	6	E	$(2/3)^4 \times 2^3$	128/81
7	5	F	$(2/3)^{11} \times 2^7$	3/2
8		F#	$(2/3)^6 \times 2^4$	1024/729
9	4	G	$(2/3)^1 \times 2^1$	4/3
10		G#	$(2/3)^8 \times 2^5$	8192/6561
11	3	A	$(2/3)^3 \times 2^2$	32/27
12		A#	$(2/3)^{10} \times 2^6$	65536/59049
13	2	B	$(2/3)^5 \times 2^3$	256/243
14	1	C	Perfect Octave	1

Table 15 duplicates Table 11, adding column 2, which numbers the notes in the scale from top to bottom. Table 15 also adds line 1 — the lower octave of B (string 9), and line 14 — the higher octave of C (string 1). In Table 15, the notes in the *išartum* tuning from Figure 51 are printed in red. (Notice that this tuning is identical to the ancient Chinese heptatonic scale, with up-generated fundamental,[71]

71 Notice that the fundamental in the *išartum* tuning, assumed here to be C, is the second fifth in a series of seven fifths. Therefore, this fundamental, C, must be up-generated from the first fifth in the series: F. Recall that an up-generated fundamental is also described in the *Guanzi*, in the construction of the pentatonic scale (see footnote 57).

shown in Figure 43.) Figure 52 shows the *sammû* tuned to the *išartum* tuning given in Figure 51. To play this tuning as a descending scale, the strings were plucked in order, from 1 to 7 (or 1 to 9 — if the two duplicate notes were included).

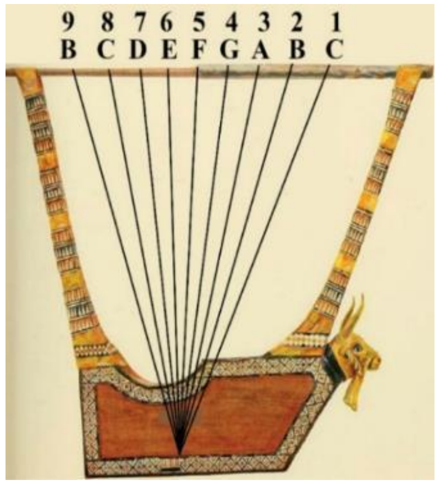

Figure 52. The *sammû*, tuned to *išartum*

To identify a pattern that we'll see repeat in the following tunings, the information given by UET VII 74 — i.e., the "tuning numbers" (for example, 2-6 is the number of *išartum* — see Table 14), the tritone position, and the string to be tightened — is summarized in line 1 of Table 16. Table 16 also lists the point of the heptagram that indicates the first note of the tuning.

Table 16. The Mesopotamian Tunings, as Described by Chapter 1 of UET VII 74 and Illustrated by Our Model

Line	1st Note Given by Heptagram Point:	String to be Tightened	Tritone Position	Tuning Numbers	Tuning Name	Modern Modal Equivalent
1	1	5	5-2	2-6	*išartum*	Ionian Mode
2	4	1	1-5	5-2	*qablītum*	Lydian Mode
3	7	4	4-1	1-5	*nīš tuḫrim*	Locrian Mode
4	3	7	7-4	4-1	*nīd qablim*	Phrygian Mode
5	6	3	3-7	7-4	*pītum*	Aeolian Mode
6	2	6	6-3	3-7	*embūbum*	Dorian Mode
7	5	2	2-6	6-3	*kitmum*	Mixolydian Mode

The next tuning described in Chapter 1 of UET VII 74 is *qablītum*:

> If the *sammû* is *qablītum* and *nīš tuḫrim* is not clear, tighten string 1 and *nīš tuḫrim* will be clear.

If our model is to correctly illustrate the tuning *qablītum*, two conditions must be met:

- The "tritone line" must lie between green numbers 1 and 5 (*nīš tuḫrim*).
- Relative to the *išartum* mode, string 5 must be tightened (i.e., six of the points of the heptagram must indicate the same notes as those indicated in Figure 51, but one point must indicate F#, rather than F.) (Note that this tuning is identical to the ancient Chinese heptatonic scale — with a fundamental that is *not* up-generated — shown in Figure 44.)

Through trial and error, it becomes apparent that only one position of the heptagram allows these conditions to be met: when point 4 indicates C, as shown in Figure 53.

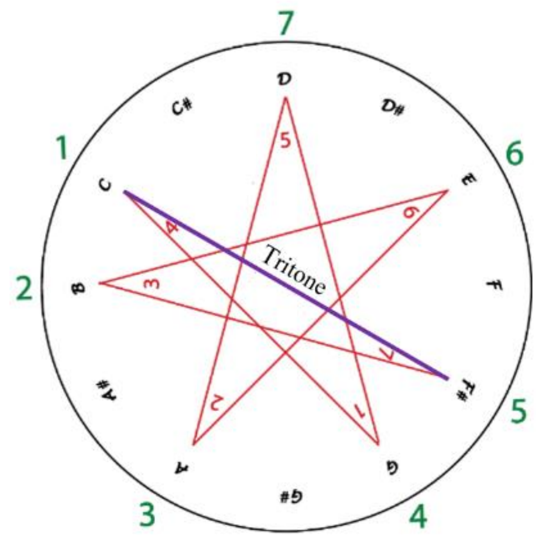

Figure 53. The *qablītum* Tuning: C, B, A, G, F#, E, D

The information for the tuning *qablītum* is summarized in line 2 of Table 16. In Figure 54, the *sammû* is shown tuned to the *qablītum* tuning given in Figure 53. To establish a pattern, two more tunings will suffice.

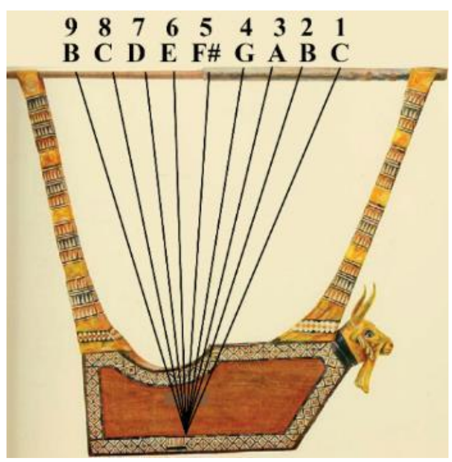

Figure 54. The *sammû*, tuned to *qablītum*

The next tuning described in Chapter 1 is *nīš tuḫrim*:

If the *sammû* is *nīš tuḫrim* and *nīd qablim* is not clear, tighten string 4 and *nīd qablim* will be clear.

As before, two conditions must be met:

- The tritone line must lie between green numbers 4 and 1 (*nīd qablim*).
- Relative to the *qablītum* tuning, string 1 must be tightened (i.e., six of the heptagram's points must indicate the same notes in Figure 53, but one point must indicate C#, rather than C.)

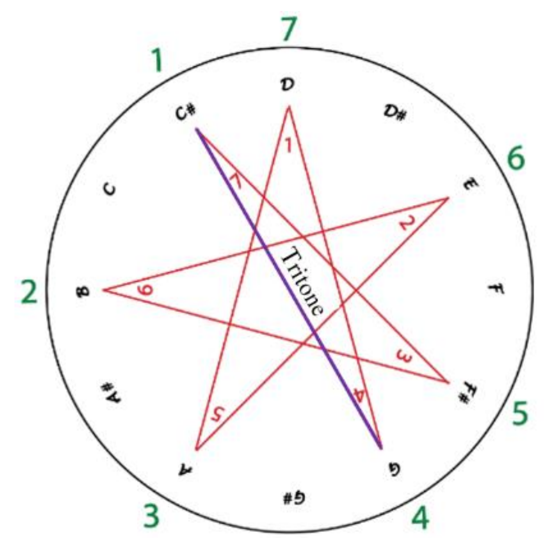

Figure 55. The *nīš tuḫrim* Tuning: C#, B, A, G, F#, E, D

As before, only one position of the heptagram allows these conditions to be met: when point 7 indicates C# (Figure 55). The information for *nīš tuḫrim* is given in line 3 of Table 16.

The next tuning in Chapter 1 is *nīd qablim*:

If the *sammû* is *nīd qablim* and *pītum* is not clear, tighten string 7 and *pītum* will be clear.

As before, two conditions must be met:

- The tritone line must lie between green numbers 7 and 4 (*pītum*).

- Relative to the *nīš tuḫrim* tuning, string 4 must be tightened (i.e., six of the points must indicate the same notes in Figure 55, but one point must indicate G#, rather than G.)

As before, only one position of the heptagram allows these conditions to be met: when point 3 indicates the note C#, as shown in Figure 56. The information for *nīd qablim* is given in line 4 of Table 16. Following a similar rationale in constructing the remaining three tunings will generate the data in lines 5, 6, and 7 of Table 16.

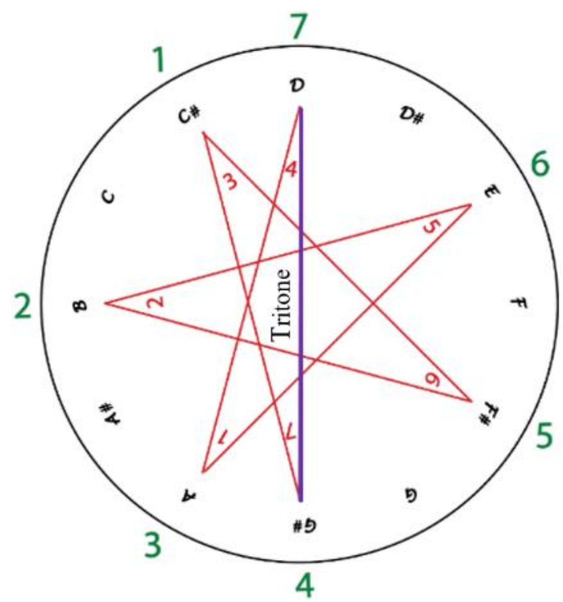

Figure 56. The *nīd qablim* Tuning: C#, B, A, G#, F#, E, D

Let's look, now, for a pattern in Table 16. Notice that every vertical line of numbers, when reading from the bottom of the table, up, is an inversion of the sequence 4,1,5,2,6,3,7. Also notice that every horizontal line of numbers, when starting from the left and removing numbers that are duplicated, is also an inversion of this sequence (or its first four numbers). Because of the multiple repetitions of this sequence, it is possible to create a simple mnemonic device that summarizes the information given in Table 16 (and, consequently, the entire "re-tuning cycle" on UET VII 74).

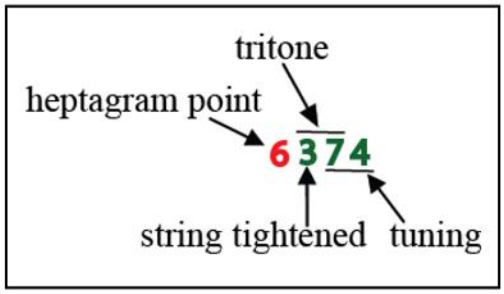

Figure 57. A Mnemonic Device

For example, to construct the next tuning in the cycle, *pītum*, we turn the heptagram so that point 6 (Table 16, column 2, line 5) indicates the first note of the tuning. To begin, we write down this number (6), as shown in Figure 57. Now we write after it the next three numbers in the sequence: in this case, numbers 3, 7 and 4. The first green number (3) gives the string to be tightened; the first two green numbers (3-7) give the position of the tritone; the last two green numbers (7- 4) give the tuning: *pītum*. This mnemonic device will work for every tuning.

It becomes apparent, therefore, that the re-tuning cycle can be mastered simply by memorizing the seven tuning/interval names and the sequence 4,1,5,2,6,3,7. Moreover, the re-tuning cycle is illustrated *exactly* by the irregular heptagram we derived earlier, using the ancient Chinese up-and-down principle — even to the degree that listing the points of the heptagram that indicate the first note of each consecutive tuning generates the reverse order of the sequence 4,1,5,2,6,3,7 (Table 16, column 2).

The construction of the diatonic tunings given above is the same as those given by other scholars in the field. For example, Figure 58 shows the tunings described in Chapter 1 of UET VII 74, as constructed by L. Crickmore. (Similar constructions are given by Gurney [72] and Friberg [73].)

Notice that the pitches given by Crickmore for the first four tunings are identical to those given by our model in Figures 51, 53, 55, and 56. If we were to construct the next three tunings with our model, the notes generated would duplicate those given in Figure 58.

As shown in Figure 58, in each subsequent tuning a new pitch is created. By the sixth tuning,

72 Gurney, "Babylonian Music Again," 103.

73 Friberg, "Seven-Sided Star Figures," 139.

embūbum, all twelve chromatic pitches have been generated. Listing the pitches in the order that they are generated, we see that they are consecutive fifths: F#, C#, G#, D#, A#. The seventh tuning, *kitmum*, generates one last fifth: E#. This E# is the Pythagorean-comma-equivalent of the note F in the very first tuning (*isartum*).

We see, therefore, that, just as with the *sanfen sunyi* method, the re-tuning cycle on UET VII 74 generates twelve pitches from a line of consecutive fifths. Yet, unlike the ancient Chinese texts, the cuneiform tablets that describe the Mesopotamian tonal system make no mention of a mathematical method for generating these pitches, or even of the fact that twelve pitches underlie this diatonic system. Instead, the instructions on UET VII 74 simply generate the twelve pitches while re-tuning the *sammû* through a cycle of seven diatonic modes. The question arises, therefore: which has chronological precedence — the diatonic scale or the circle (or spiral) of fifths?

No.	1		2		3		4		5		6		7	Tritone	Re-tuning
Name	*išartum*														
	c″*		b′		a′		g′		f′		e′		d′	5-2	5# **
		s		t		t		t		s		t			
Name	*qablītum*														
	c″		b′		a′		g′		f#′		e′		d′	1-5	1#, 8#
		s		t		t		s		t		t			
Name	*nīš tuḫrim* (also read *nīš* GABARI)														
	c#″		b′		a′		g′		f#′		e′		d′	4-1	4#
		t		t		t		s		t		t			
Name	*nīd qablim*														
	c#″		b′		a′		g#′		f#′		e′		d′	7-4	7#
		t		t		s		t		t		t			
Name	*pītum*														
	c#″		b′		a′		g#′		f#′		e′		d#′	3-7	3#
		t		t		s		t		t		s			
Name	*embūbum*														
	c#″		b′		a#′		g#′		f#′		e′		d#′	6-3	6#
		t		s		t		t		t		s			
Name	*kitmum*														
	c#″		b′		a#′		g#′		f#′		e#′		d#′	2-6	2#, 9#
		t		s		t		t		s		t			

* ″ denotes higher octave   ** # denotes tightening (or sharping) by one semitone

Figure 58. Figure 5 from L. Crickmore, "New Light on the Babylonian Tonal System," (ICONEA, 2008): 14

13. WHICH CAME FIRST, THE DIATONIC SCALE OR THE CIRCLE OF FIFTHS?

As previously mentioned, if the points of the heptagram on CBS 1766 are assumed to be equidistant, they cannot indicate twelve positions — and therefore twelve pitches — on a circumscribed circle. *It is only the irregular heptagram derived using the ancient Chinese up-and-down principle that, when superimposed on a circle of twelve chromatic notes, can exactly illustrate the re-tuning cycle described on UET VII 74*. As noted by J. C. Franklin, this discovery is attributed to the author:

> De Rose's hypothesis, … offers to tie together the best of the foregoing insights and intuitions … but [is] able to do the job with much greater precision and practicality. The essential point is that her star has a kind of asymmetry. It is derived not from an equal, seven-part division of an encompassing circle, but a 12-part division, from which the stars' points are chosen to allow one to track the positions of tritones and semitones—*immediately, visually and physically*—with no prior knowledge necessary, and so as to allow the harmonic space of the Re-tuning Cycle to be neatly charted within the geometry of a circle. [74]

Franklin acknowledges that the irregular heptagram derived earlier illustrates the re-tuning cycle described on UET VII 74, but he describes the points of the heptagram as being "chosen," implying that the heptagram is an arbitrary human construct. The reason for this is that Franklin doesn't believe that the relationship of the heptagram to the mathematics of music could have been discovered by the Mesopotamians until *after* the re-tuning cycle was already in place — for Franklin believes that use of the diatonic scale must have predated any knowledge of harmonic ratios.

In other words, Franklin's position is similar to that of Bagley (see footnote 36): while Bagley believes that the chromatic scale was first generated by transposing the *pentatonic* scale, Franklin

74 J. C. Franklin, "Examination of Sara de Rose's Proposed Link between CBS 1766 and UET VII 74," Unpublished paper presented at the University of Würzburg, Germany, at a 2018 conference entitled "Tonal Systems and Music Notation in Mesopotamian and Related Cultures."

believes that it arose through transpositions of the *diatonic* scale — and that UET VII 74 supports this hypothesis because it doesn't refer directly to mathematical ratios. Consequently, for Franklin, the irregular heptagram simply *happens* to illustrate a pre-existing auditory experience:

> The tuning instructions of UET VII 74 itself, as presented, rely upon ear, eye, and fingers to determine and resolve the "clarity" or "unclarity" of intervals. We would not say that these intervals have a "mathematical origin" just because their frequency ratios stand in epimoric relations like 3:2 and 4:3. Yes, there is a kind of underlying mathematics; but the direct experience of sound must obviously have preceded the knowledge of these numbers.[75]

Franklin believes that an awareness of harmonic ratios could not have predated the writing of UET VII 74 (circa 1800 BC). Yet there is textual evidence that suggests that the Mesopotamians had this knowledge in the late third millenium BC. King Shulgi (2094 BC – 2047 BC), ruler of the city of Ur, is recorded as having a masterful command of music. An account of his skills is given in what is now called a "royal praise-hymn" — an autobiographical poem full of royal self-praise:

> I, Shulgi, king of Ur, have ... devoted myself to the musician's art.... When I fix the frets on the lute ... I never damage its neck; I have established procedures for raising and lowering its intervals.... Even if they bring to me ... a lute ... that I have not heard before, when I strike it up I make its true sound known.... If in tuning I tighten, loosen or set [sc. the strings], they do not slip from my hand....[76]

Shulgi refers to the lute (Figure 59), an instrument that first appeared in Mesopotamia circa 2300 BC. Unlike the harp and lyre, the lute has a fret board. Shulgi describes fixing the frets of the lute,

75 Franklin, "Examination of Sara de Rose's Proposed Link."

76 *Shulgi B*, lines 154–174. (Trans. adapted from ETCSL [Electronic Text Corpus of Sumerian Literature] 2.4.2.02, partially on the basis of text and commentary in Krispin (1990) as given by Franklin in *Kinyras: The Divine Lyre* (2015): 34.)

which suggests that the frets were moveable. How does Shulgi determine the placement of the frets? — by "raising and lowering its intervals." By this phrase, Shulgi is not referring to the tuning procedure described on UET VII 74, for a description of the tightening and loosening of the strings required by the re-tuning cycle comes later in the praise-hymn: "if in tuning I tighten, loosen or set [sc. the strings], they do not slip from my hand." "Raising" is associated with the word "up," and "lowering" with the word "down." Is Shulgi, in referring to the "procedures for raising and lowering its intervals," describing the up-and-down principle?

Figure 59. Mesopotamian Lute, circa 1800 BC (Oriental Institute, University of Chicago)

As was shown in Figure 24, the position of the frets that mark where a depressed string sounds the pitches generated by the up-and-down principle can be derived by a simple folding exercise, using whatever materials are at hand: paper, string, etc. In other words, to generate such a fretboard no multiplication or division need be done: the folded material itself makes these calculations. If this method was known to the Mesopotamians, even the simple mathematics underlying the up-and-down principle could have been avoided, and the heptagram on CBS 1766 (which is derived by simply keeping a tally of the 2/3 folds and the doubling folds) could have been generated — *without previous knowledge of the diatonic scale*.

It is now the general consensus that the Mesopotamian tonal system was transmitted to Greece, where the seven Mesopotamian modes were given Greek names. As further evidence of this transmission, J. Friberg, an expert in Mesopotamian mathematics, notes that some of the terms used by Plato in *Timaeus* to describe the method of calculating the ratios of the notes of the diatonic scale have a distinctly Mesopotamian flavor:

> Strange terms in this passage [*Timaeus* (35b-36b)], borrowed from Pythagorean music theory, are "hemiolic," from Greek hemiolios ("a half and a whole," meaning 1 1/2) [and] "epitritic", from Greek epitritos ("a third more", meaning 1 1/3.... [S]uch expressions are not unusual in Old Babylonian mathematical texts....[77]

Notice that one of these mathematical expressions — "a third more" (or "epitritic") — is used in the *Lǚshì chūnqiū* ("the addition of 1/3 of its value") to describe an up-generation. As Friberg describes, this expression is "not unusual in Old Babylonian mathematical texts" (although in mathematical, not musical, contexts). Is this further evidence for a mathematical basis of the Mesopotamian tonal system and for a transmission of this system, both east and west?

We saw earlier that the shape of the heptagram is not a "choice": *it is a geometric expression of a mathematical algorithm that generates a twelve-tone scale using the fractions 1/2 and 2/3*. Moreover, when the heptagram is derived, *there is no suggestion of a seven-note scale*. Instead, a *twelve-tone* scale is generated and, in so doing, *a sequence of seven numbers*: 4,1,5,2,6,3,7. This sequence indicates the number of doublings required to move each consecutive fifth into the octave of the scale under construction, *but it has, as yet, no diatonic application*.

It is only *after* the twelve-tone scale has been created that the sequence 4,1,5,2,6,3,7 can have a diatonic application. For, as we have seen, when superimposed on the circle of fifths this sequence selects the notes in the diatonic scale (Figure 43) — as do the points of the irregular heptagram derived earlier, when it is superimposed on the chromatic circle (Figure 49).

But how is it possible that the sequence 4,1,5,2,6,3,7 has these two applications? How can it, on

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77 Friberg, "Seven-Sided Star Figures," 144.

one hand, indicate the number of doublings required to create a *twelve-tone* scale generated by the up-and-down principle and, on the other hand, select the *seven* notes of the diatonic scale from the circle of fifths (and, consequently, provide the pattern upon which the Mesopotamian re-tuning cycle is based)? The answer is: simple mathematics.

With respect to the first application, the sequence is generated because twelve fifths span seven octaves. In "doubling" these fifths into a single octave to create a twelve-tone scale, the sequence is generated. With respect to the second application, remember that the notes of the diatonic scale are consecutive fifths and that *the interval between each fifth is seven semitones* (Figure 45). Therefore, in selecting the notes of the diatonic scale from the twelve-tone scale, leaps of *seven* semitones are made through octaves divided into twelve semitones — causing, once again, the sequence 4,1,5,2,6,3,7 to be generated.

In other words, both instances in which the sequence is generated have to do with the interplay of the numbers 7 and 12: it is the "gearing" of cycles of 7 and cycles of 12 that generates the sequence. Yet for Franklin, convinced of the precedence of the diatonic scale, this mathematical underpinning seems arbitrary:

> I worry that these mathematical procedures are arbitrary. They seem to work, simply because they describe real harmonic phenomena. There is nothing about the Re-tuning Cycle, as set forth in UET VII 74, that requires such knowledge.[78]

It is the ancient Chinese tonal system that offers a solution to Franklin's query. For, as we have seen, the Chinese used the *sanfen sunyi* method — which they identified as having a mathematical origin — to generate scales having a varying number of notes. For example, according to the *Zuo zhuan* (fourth century BC), five-note, six-note and seven-note scales were in use. Yet the *Zuo zhuan* gives no special importance to the seven-note scale. In fact, it is described as subordinate to the five-tone scale:

---

78 Franklin, "Examination of Sara de Rose's Proposed Link."

"The seven-note scale and the six-pitch scale subordinate to the five-tone scale. [七音，六律，以奉五聲.]" [79]

In other words, the ancient Chinese used the *sanfen sunyi* method, from which the sequence 4,1,5,2,6,3,7 is generated, but gave no innate importance to the seven-note diatonic scale. Instead, for them, it was the diatonic scale that was the arbitrary "choice" — not the mathematics that form the underlying structure of the twelve-tone system, from which all scales were derived.

It can be argued, of course, that the fact that the heptagram on CBS 1766 appears to be related to the *sanfen sunyi* method does not prove transmission of the knowledge of harmonic ratios from Mesopotamia, for the laws of harmonics could have been discovered independently in China. Yet, as we have seen, there is strong evidence that both stringed instruments and the up-and-down principle did, in fact, originate in Mesopotamian and were transmitted to China.

It is important to note that the hypothesis that the Mesopotamians had knowledge of the up-and-down principle can, in fact, answer some of the unresolved questions regarding the nature of the Mesopotamian tonal system. For example, if the Mesopotamians were aware of this principle, it could explain why some of the Near Eastern instruments we looked at earlier had fewer than seven strings. For perhaps the Mesopotamians, like the Chinese, applied the up-and-down principle to construct scales having fewer than seven notes. But if this is the case, why does the re-tuning cycle on UET VII 74 describe, exclusively, seven-note, diatonic scales?

As we've seen, every scale generated by the up-and-down principle is constructed from a series of consecutive fifths. As a result, these scales are compatible with each other: the five-note scale is nestled within the six-note scale, which is, in turn, nestled within the seven-note scale. In other words, in terms of the up-and-down principle, a scale having less than seven notes is a *subset* of the diatonic scale.

For example, relative to the circle of fifths, the pentatonic scale is contained within the diatonic scale (Figure 60). Similarly, the notes of the pentatonic scale are selected from the chromatic circle by five of the seven points of the irregular heptagram (Figure 61).

79 *Zuo Zhuan* (左传) Year Twenty-Five of Lord Zhao (昭公二十五年), trans. C. Y. Chen, *Early Chinese Work in Natural Science* (Hong Kong University Press, 1996), 108.

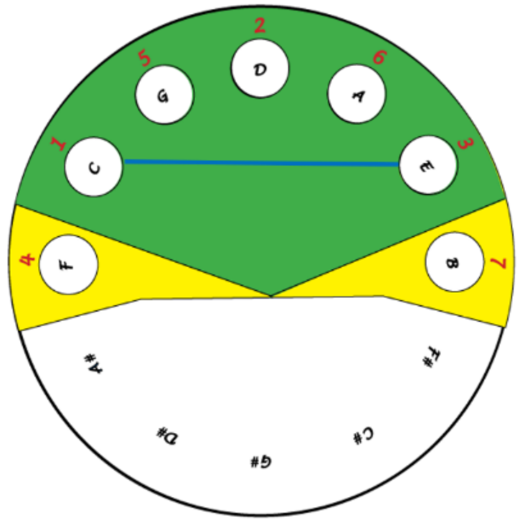

Figure 60. The Pentatonic Scale contained within the Diatonic Scale (Circle of Fifths)

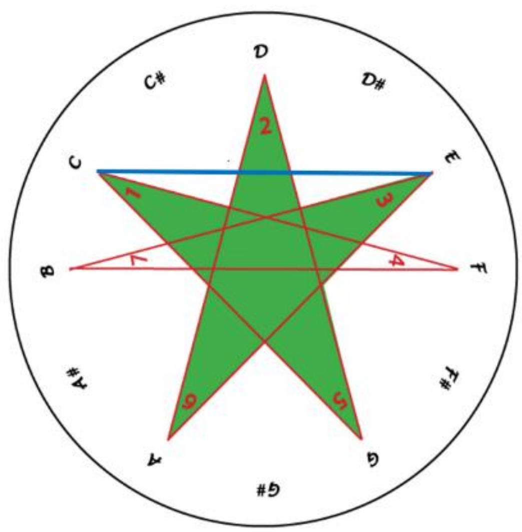

Figure 61. The Pentatonic Scale: Pentagram contained within Heptagram (Chromatic Circle)

In fact, the sequence 4,1,5,2,6,3,7 with the numbers 4 and 7 removed — 1,5,2,6,3 [80] — functions as a mnemonic device that describes a *pentatonic* re-tuning cycle, exactly indicating the order of the

80 Note: the sequence 1,5,2,6,3 is the equivalent of the sequence 1,4,2,5,3 (shown in Figure 42 and column 3 of Table 2), because notes 5 and 6 are the same as notes 4 and 5, respectively.

strings to be re-tuned, the point of the heptagram that indicates the first note of each mode, etc. Consequently, *the diagram on CBS 1766 can be used to construct not only the diatonic modes, but the pentatonic modes*.[81]

The main difference between these two re-tuning cycles is that, with respect to the diatonic scale, the "not clear" interval is a tritone (six semitones) while relative to the pentatonic scale the "not clear" interval is a minor sixth (eight semitones). In other words, relative to the diatonic scale, the "not clear" interval is made "clear" by *widening* the interval by one semitone, while relative to the pentatonic scale the "not clear" interval is made "clear" by *narrowing* the interval by one semitone. The "not clear" interval of the pentatonic scale is shown in Figure 60 and Figure 61 by the blue lines, which show the span of a minor sixth (for example, from E to C). It is interesting, therefore, that three of the intervals on tablet CBS 10996 (of those not printed in red in Table 14) are, in fact minor sixths (or the inverse: major thirds). Figure 62 shows these three intervals from CBS 10996 — *isqum* (1→3), *serum* (7→5), and *s/zerdum* (4→6) — relative to the scale of C Major. In each case, the interval spans one third of the circle, as shown by the blue lines in Figure 62.

From this, we see a general rule: on both the circle of fifths (Figure 60) and on the chromatic circle (Figure 61), the interval of a minor sixth (or major third) — which is the size of the intervals *isqum*, *serum*, and *s/zerdum* — spans one third of the circle. Therefore, on either circle, two notes separated by a minor sixth (or major third) are found at two points of an equilateral triangle. Rotating the triangle shows all possible minor sixth (or major third) intervals (Figure 63).

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81 Incidentally, if we perform the up-and-down principle using the fractions 1/2 and 3/4, rather than 1/2 and 2/3, a twelve-tone scale is also generated. In this case, the pitches are consecutive fourths, rather than consecutive fifths, but where twelve fifths span seven octaves, twelve fourths span *five* octaves. When these twelve fourths are reduced into a single octave to create a twelve-tone scale, the sequence 1,3,5,2,4 is generated. This is the sequence shown in Figure 42, and in the instruction from the *Lǚshì chūnqiū* (see Table 2, column 3) — but in reverse order. Why is the sequence reversed? Because the circle of fourths is, in fact, the circle of fifths in reverse order: each note in a *counter*-clockwise direction is the fourth of the note before it. Also, recall that the heptagram we derived earlier can be used as part of a "computer" that calculates the string lengths of the notes generated when creating a twelve-tone scale from perfect fifths. Similarly, the mathematical generation of a twelve-tone scale constructed from consecutive fourths can be illustrated, geometrically, by the circle of fifths/fourths and the five-pointed irregular heptagram shown in Figure 61. (See footnote 80 to understand the numbering on the points of the pentagram in Figure 61.)

The remaining four intervals on CBS 10996 (of those not printed in red in Table 14) are major sixths (or the inverse: minor thirds) and are shown by the orange lines in Figure 62. As shown in Figure 64, two notes that span this interval are separated — on both the chromatic circle and the circle of fifths — by one quarter of the circle and are, therefore, located at two corners of a square. Rotating the square shows all possible major sixth (or minor third) intervals.

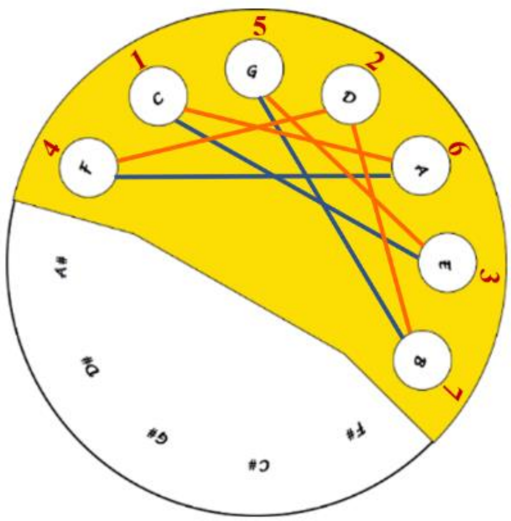

Figure 62. Blue lines: minor sixth (or major third) intervals. Orange lines: major sixth (or minor third) intervals

Recall that the seven intervals from CBS 10996 that are printed in red in Table 14 generate the sequence 1,5,2,6,3,7,4 and, therefore, ultimately, the heptagram on CBS 1766. We now see that the other seven intervals from CBS 10996 also suggest rotating shapes: a triangle and a square. In other words, there is an underlying geometry inherent in the Mesopotamian tonal system — a geometry that, it is proposed here, could not have arisen without knowledge of the circle of fifths.

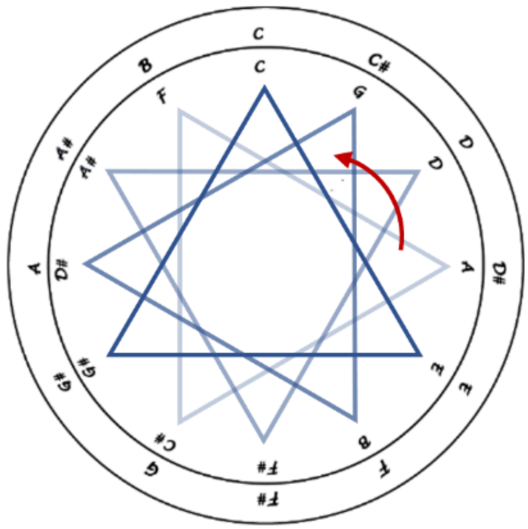

Figure 63. Triangle shows major third/minor sixth interval on Chromatic Circle/Circle of Fifths

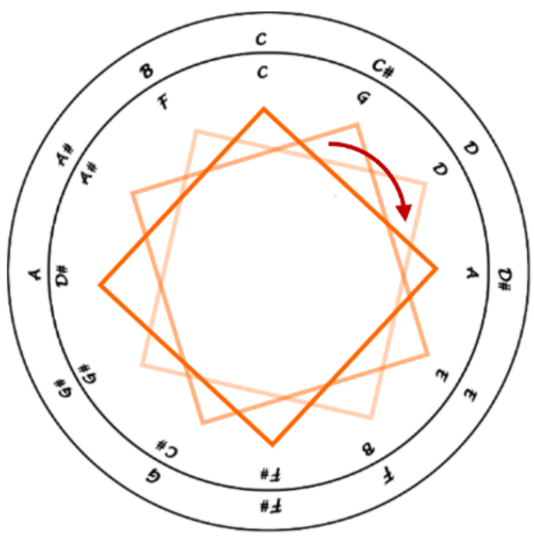

Figure 64. Square shows minor third/major sixth interval on Chromatic Circle/Circle of Fifths

14. SUMMARY

The Mesopotamian tonal system was understood, by the people who used it, to be expressible both numerically and geometrically. The primary numeric pattern, the sequence 4,1,5,2,6,3,7, is found embedded in the ancient Chinese up-and-down principle, suggesting that this mathematical procedure was transmitted from the Near East. But what of the geometric patterns? Was knowledge of the relationship between music and geometry also transmitted?

In fact, the nomenclature of the Marquis Yi bells is derived from the relationship between major thirds (or minor sixths) — the span of the three intervals listed on CBS 10996, which we have seen to be illustrated, relative to the circle of fifths/chromatic circle, by two points of an equilateral triangle (Figure 63). As we saw earlier, the bells give the names of the pitches of the chromatic scale, but twelve distinct names are not used. Instead, there are four core names: *gōng*, *zhǐ*, *shāng*, and *yǔ*. Each of these names is used alone, with the suffix *jué*, or with the suffix *zeng* — thereby generating twelve distinct names. These suffixes indicate the addition of major thirds: *jué* adds one major third; *zeng* adds two major thirds (see Table 8). Because the pitches *gōng*, *zhǐ*, *shāng*, and *yǔ* are consecutive notes on the circle of fifths (for example, relative to the C pentatonic scale, they are C, G, D and A, respectively), rotating the red triangle in Figure 65 to any of its four possible positions illustrates the generation of the nomenclature of the chromatic scale of the Marquis Yi bells "by adding major thirds to each of the four monosyllables [*gōng*, *zhǐ*, *shāng*, *yǔ*]."[82]

82 Bagley, "Prehistory of Chinese Music," 67.

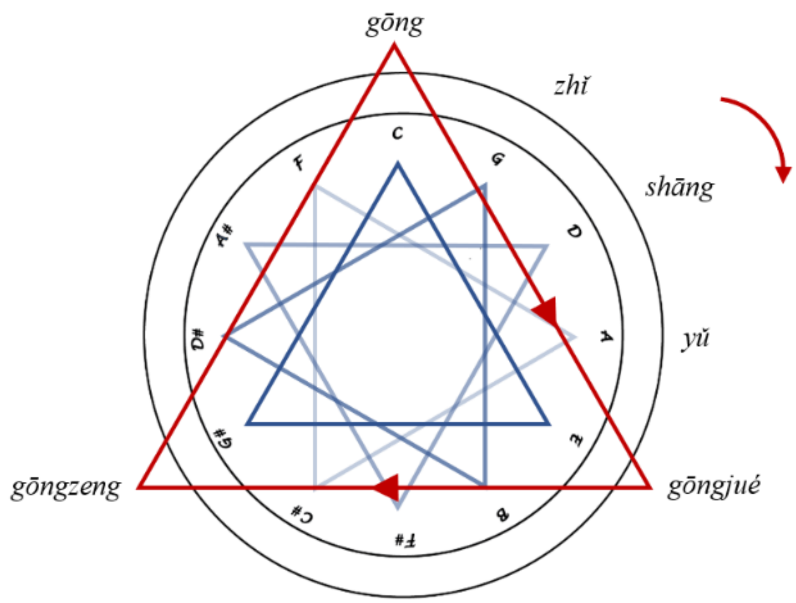

Figure 65. The geometry underlying the *jué/zeng* nomenclature of the Marquis Yi Bells

The importance of the intervals of the major third and minor third (which, together, make up seven of the fourteen intervals listed on CBS 10996) to the ancient Chinese is also shown by the fact that the pitch produced at the alternate strike-portion on each Marquis Yi bell is either a major third or minor third above the primary pitch.

Nevertheless, it must be admitted that explicit knowledge of the relationship between geometry and music is not described in any ancient Chinese text. Western textual evidence of knowledge of this relationship is only slightly less scanty.

Pythagoras (570–495 BC), who is often (erroneously?) cited as the first to study the mathematics of music, is credited with the saying "there is geometry in the humming of the strings…."[83] Yet many historians believe that Pythagoras's musical knowledge was learned in Babylon. For according to Iamblichus (245–325 AD), when Pythagoras was in Egypt he was "taken captive … [and] brought to Babylon. Here the Magi instructed him in their venerable knowledge and he arrived at the summit of arithmetic, music, and other disciplines…."[84] Iamblichus describes that Pythagoras returned to Greece

83 *The Houghton Mifflin Dictionary of Biography* (2003), 1250.

84 Iamblichus, *The Life of Pythagoras, Ch. IV*, trans. T. Taylor (Theosophical Pub. House, 1918), 10.

when he was fifty-six years of age. According to this timeline, Pythagoras would have been in Mesopotamia less than a century after some of the tablets that relate to the Mesopotamian tonal system were written.

Let's return to our discussion of whether the Mesopotamians used scales other than the diatonic. In terms of using scales having fewer than seven notes, recall that the means of creating such scales using the up-and-down principle is virtually identical to the process described on UET VII 74. For regardless of the number of notes, there is always an "unclear" interval between the first and last fifth. Yet the musician need never know how to tune this interval, but simply to re-tune it as a fifth. Therefore, considering the ease with which the re-tuning cycle on UET VII 74 could have been modified to generate scales having fewer than seven notes, coupled with the fact that many instruments are depicted, in the ancient Near East, as having fewer than seven strings, it is reasonable to imagine that the Mesopotamians used scales having fewer than seven notes.

On the other hand, scales having more than seven notes were probably not used. For on UET VII 74, strings 8 and 9 are tuned with strings 1 and 2, suggesting that on instruments having more than seven strings, strings 8 and above were octaves of strings 1 to 7.

Instruments that support many strings must have greater structural integrity than those that support fewer strings — but this can make them large and difficult to carry. We must consider, therefore, that the angular harps found in Xinjiang arrived there because they were small, *and that being small necessitated that they were strung with a limited number of strings*. Yet, as we have seen, this does not necessarily indicate that they had lost their connection with Mesopotamian tuning theory. Instead, it is proposed here that the Mesopotamian tonal system, like the ancient Chinese system, was capable of generating scales having a varying number of notes, most notably pentatonic and diatonic scales, and that the mathematical method of generating these scales — the up-and-down principle, from which the sequence 4,1,5,2,6,3,7 is derived — was widely understood.

In summary, we have seen that this sequence has two applications: (1) it operates as part of a "computer" that calculates the string lengths of the notes in a twelve-tone scale; (2) it arranges the notes of the diatonic scale as seven consecutive fifths. It is proposed here that the use of the diatonic scale as the basis of the Mesopotamian tonal system (and the consequent appearance of the sequence 4,1,5,2,6,3,7 on CBS 1766, CBS 10996, and UET VII 74) was an *intentional choice* that came *after* the discovery of the mathematical origin of the sequence. For whereas the use of a random selection of seven pitches to construct a scale could be interpreted as an audible preference, the use of seven consecutive fifths — which, in order to create a scale, must be moved into a single octave — points to an underlying awareness of simple musical ratios.

Although the seven-number sequence has seven inversions, we have noted only two inversions: 4,1,5,2,6,3,7 and 1,5,2,6,3,7,4. Why? Because these two inversions are the only ones generated by the up-and-down principle: the first when a diatonic scale is derived from a fundamental that is up-generated (Figure 43); the second when a diatonic scale is derived from a fundamental is *not* up-generated (Figure 44). Yet, even before we derived the diatonic scale, we saw these two inversions appear when creating a twelve-tone scale. For notice that in column 4 of Table 4, Table 5, and Table 6, the sequence is begun, in each case, with the number 4. Yet, when we extended the scale downward (Table 7) the sequence began (in column 3, line 1) with the number 1. We see, therefore, that these two inversions are, in fact, more fundamental than the other five.

In the next part of this paper, we will focus exclusively on the inversion 4,1,5,2,6,3,7. Why? Because, as we'll now see, there is evidence that the sequence 4,1,5,2,6,3,7 was used, in the West, in non-musical, esoteric applications, suggesting that the use of this sequence to create the modes of the diatonic scale was, perhaps, part of a wider tradition.

PART TWO:

MUSICO-COSMOLOGIES OF THE EAST AND WEST AND THEIR RELATIONSHIP TO CORRELATIVE THINKING

15. MUSICO-COSMOLOGY IN MESOPOTAMIA AND THE WEST

As we have seen, there is strong evidence that ancient Greece, which is often considered the birthplace of Western civilization, inherited its musical system from Mesopotamia. Therefore, let's begin our discussion of Western musico-cosmology by looking at sources from Mesopotamia.

The number seven appeared frequently in the Mesopotamian magical rituals. Often "seven-magic" was used in conjunction with music and musical instruments. This is probably connected, as J. C. Franklin suggests,[85] to the fact that the Mesopotamian tonal system was heptatonic. An example that Franklin cites is from a tablet, dated 2400 BC, that describes offerings given to seven *balangs*: "seven liters of oil and seven liters of dates for the seven *balangs*."[86]

W. Heimpel[87] identifies the *balang* as the arched harp, because a pictogram that resembles an arched harp (Figure 66), used in ancient texts, was replaced, in later cuneiform texts, with the word *balaĝ* — which is rendered phonetically as *balang*. In the text cited by Franklin, seven *balangs* are deified, receiving offerings of oil and dates.

Franklin also gives an example of music seven-magic from *The Cursing of Agade* (2047–1750 BC), a text describing the god Enlil's destruction of the city of Akkad. In an attempt to appease Enlil and restore order, seven *balangs* are ritually arranged by the chief lamentation singer, who, "for seven days and seven nights put in place seven *balangs*, like the firm base of heaven."[88]

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85 John C. Franklin, "Lyre Gods of the Bronze Age Musical Koine," *Journal of Ancient Near Eastern Religions* 6.2 (2006): 58.

86 TSA 1 ix:12–14: dated to end of first dynasty of Lagash, p. IX., as given by J. C. Franklin in *Kinyras: The Divine Lyre* (Hellenic Studies Series 70. Washington, DC: Center for Hellenic Studies), 41.

87 https://chs.harvard.edu/CHS/article/display/6337.balang-gods-wolfgang-heimpel

88 Electronic Text Corpus of Sumerian Literature 2.1.5, lines 196–204 (as given by J .C. Franklin in *Kinyras: The Divine Lyre* (2015), 41.)

Figure 66. Harp Pictogram (3000 BC) (Oriental Institute, University of Chicago)

The number seven also had importance in Mesopotamian cosmology. This is no doubt because there are only seven bodies that appear to move independently, yet regularly,[89] and that can be seen with the naked eye: the sun, the moon, Mercury, Venus, Mars, Jupiter and Saturn.

We know from cuneiform texts that as early as 1800 BC the Babylonians identified these seven bodies as "gods" and, also, as "planets." For example, the "Great Star List" enumerates: "The moon and the sun, Jupiter, Venus ... Saturn, Mercury ... Mars" and concludes with the summary statement "Seven planets."[90] We also know that detailed planetary observations — of Venus, for example[91] — were made as early as the seventh century BC.

Yet even before the making of detailed observations, the Mesopotamians pictured the heavens as having seven levels. For example, Sumerian texts from the late second millennium BC declare that "the heavens are seven, the earths are seven."[92]

According to historian of religion M. Eliade, the Mesopotamian fascination with the number seven influenced the mythologies and cosmologies of indigenous cultures as far afield as Central Asia. For Eliade explains that although the archaic belief that people can ascend to the sky is a universal phenomenon, found in all cultures, the idea that this ascent is accomplished by climbing through *seven*

89 Although comets were observed in the ancient world, proof of their periodicity is relatively recent. (Halley, 1705)

90 Neo-Assyrian; ll. 242–244, trans. Koch-Westenholz 1995: 200–201

91 Tablet 63 from the series of tablets known as *Enuma Anu Enlil*.

92 Wayne Horowitz, *Mesopotamian Cosmic Geography* (Winona Lake, IN: Eisenbrauns, 1998), 208.

levels shows a Mesopotamian influence: "The identification of the seven-branched Cosmic Tree with the seven planetary heavens is certainly due to influences from Mesopotamia."[93] In documenting this widespread influence, Eliade observes that "the conception of seven heavens is even found throughout southeastern Siberia," citing the following examples:[94]

- "The Cosmic Pillars of the Ostyak have seven incisions."
- "The Vogul believe that the sky is reached by climbing a stairway of seven stairs."
- "The Altaic shaman climbs a tree or a post notched with seven or nine *tapty*, representing the seven or nine celestial levels."

According to Eliade, Mesopotamia also influenced the mythologies and cosmologies of the West. As an example, Eliade cites a text from Origen (184–253 AD) that gives a description from Celsus, a Greek philosopher of the second century AD. (Celsus's original writings are lost.)

> In the mysteries of Mithras ... there is a representation of ... the planets, and of the passage of the soul through these. The representation is of the following nature: There is a ladder with lofty gates.... The first gate consists of lead, the second of tin, the third of copper, the fourth of iron, the fifth of a mixture of metals, the sixth of silver, and the seventh of gold. The first gate they assign to Saturn, indicating by the 'lead' the slowness of this star; the second to Venus, comparing her to the splendour and softness of tin; the third to Jupiter, being firm and solid; the fourth to Mercury, for both Mercury and iron are fit to endure all things, and are money-making and laborious; the fifth to Mars, because, being composed of a mixture of metals, it is varied and unequal; the sixth, of silver, to the Moon; the seventh, of gold, to the Sun — thus imitating the different colours of the two latter." [95]

93 Mircea Eliade, *Shamanism: Archaic Techniques of Ecstasy* (Princeton: Princeton University Press, 2004), 274.

94 Eliade, *Shamanism*, 275.

95 Origen, *Contra Celsum*, Book VI, Chapter XXII, trans. Philip Schaff, in *Ante-Nicene Fathers*, vol. 4.

In *Shamanism: Archaic Techniques of Ecstasy*, Eliade cites the above quotation, but, for our purposes, he cuts it too short. For Origen continues:

> [Celsus] next proceeds to examine the reason of the stars being arranged in this order … [and gives] … musical reasons … quoted by the Persian theology; and to these, again, he strives to add a second explanation, connected also with musical considerations."[96]

To understand these "musical reasons" we must first understand how Celsus and his contemporaries, relying on earlier Babylonian and Greek knowledge, pictured the universe. For a naked-eye observer on earth, the sun, the moon, and the planets all appear to move through the same narrow celestial band — what, in the West, we call the ecliptic. The ecliptic was first divided into twelve constellations — which the Greeks later called the "zodiac" — by the Babylonians, circa 500 BC. Yet documentation of one of the earliest zodiac constellations — the goat-fish (Capricorn) — is found on Mesopotamian cylinder seals as early as 2100 BC.

Because the constellations of the zodiac are virtually unchanging, they provide a backdrop against which the movements of the planets can be tracked. What the observer notices is that the sun, the moon, and the planets all appear to move along the ecliptic at different speeds.

The moon appears to move most quickly through the constellations of the zodiac — taking one month to complete the full circuit — while Saturn, slowest moving of the visible planets, takes almost thirty years to return to the same constellation. Because the earth seems stationary, the sun takes on the relative motion of the earth — appearing to move more slowly than Venus, but more quickly than Mars. Arranging these seven classical planets, from fastest to slowest, in orbits (or "spheres") that are increasingly more distant from earth creates the model shown in Figure 67.

96 Origen, *Contra Celsum*, Book VI, Chapter XXII, trans. Philip Schaff.

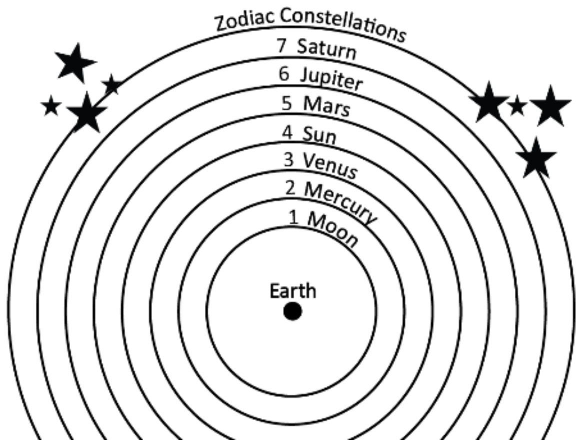

Figure 67. Ptolemaic Model of the "Universe"

To date, there is no proof that the Mesopotamians conceptualized this arrangement of the seven classical planets. However, the Alexandrian astronomer Ptolemy (100–170 AD), to whom this model is generally attributed, relied on the work of his Greek predecessor Hipparchus (190–120 BC), who is known to have had access to Babylonian knowledge.[97] A slightly different model [98] is described by Plato (428–347 BC).[99]

Figure 68 shows the Mithraic ladder described earlier by Origen, with each rung labelled with the name of the associated classical planet. Figure 68 also adds the sphere number of each planet, according to the model shown in Figure 67. Notice the sequence on the ladder: 4,1,5,2,6,3,7.

Origen writes that Celsus gives *two* musical reasons that account for "the stars being arranged in this order" — and this is exactly what we have seen.[100] For, remember, the sequence 4,1,5,2,6,3,7 is *first* derived when creating a twelve-tone scale using the up-and-down principle, and then *reapplied* to select

97 Asger Aaboe, *Episodes from the Early History of Astronomy* (New York: Springer Verlag, 2001), 62–65.

98 Plato describes the sun, Mercury, and Venus as moving together. This is, in fact, what is seen from earth: because the orbits of Mercury and Venus lie between the earth and the sun, Mercury and Venus appear to move with, yet circle, the sun — Mercury moving more quickly than Venus.

99 Plato, *Republic*, Book X, 616e, trans. Benjamin Jowett.

100 Incidentally, the heptagram on CBS 1766 is inscribed in two concentric circles, the reason for which is not known.

seven of these twelve notes to create the diatonic scale. Incidentally, the Latin word for ladder — *scala* — is the root of the musical term "scale."

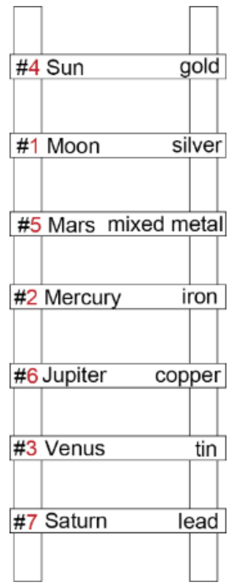

Figure 68: Mithraic Ladder

According to Celsus these musical reasons are "quoted by the Persian theology" and, as we saw earlier, artifacts do suggest that, from as early as 3000 BC, the Mesopoamian tonal system was in use in Elam (later Persia). Moreover, the Mithraic mysteries, a mystery religion centred on the god Mithras that was popular among the Roman military from the first to the fourth centuries AD, are believed, by some scholars, to have been inspired by Iranian worship of the god Mithra.

Mithra is a central divinity in the Zoroastrian religion, which was the state religion in ancient Iran for over a millennium (600 BC–650 AD). The earliest textual reference to Mithra is from an inscription at Susa that dates from the reign of Artaxerxes II (404–358 BC): "Ahuramazda, Anahita, and Mithra protect me against all evil."

As the philosopher Porphyry (234–305 AD) describes, the symbolism of the cave played an important role in Mithraic rituals, for initiates met underground, in temples or caves:

the Persians call the place a cave where they introduce an initiate to the Mysteries,

> revealing to him the path by which souls descend and go back again. For Eubulus tells us that Zoroaster was the first to dedicate a natural cave in honour of Mithras, the creator and father of all … this cave bore for him the image of the cosmos which Mithras had created, and the things which the cave contained, *by their proportionate arrangement* [author's italics], provided him with symbols of the elements and climates of the cosmos.[101]

Many of these temples have survived. For example, the "Mithraeum of the Seven Spheres," in Ostia, near Rome, contains mosaics depicting not only the divinities associated with the seven classical planets, but also the twelve constellations of the zodiac. Figure 69, for example, shows the mosaic depicting Capricorn — a goat-fish (an inheritance from Mesopotamian tradition).

Figure 69: Capricorn Mosaic, Mitreo delle Sette Sfere (circa 200 AD), Ostia, Italy

Porphyry, in describing the cave as a microcosm of the cosmos, refers also to an ancient tradition that teaches that there are two "gates" in the zodiac, through which the soul passes:

101 Porphyry, *The Cave of the Nymphs in the Odyssey*, Arethusa Monograph 1 (Buffalo: Dept. of Classics, State University of New York at Buffalo, 1969).

> Theologists therefore assert, that these two gates are Cancer and Capricorn; but Plato calls them entrances. And of these, theologists say that Cancer is the gate through which souls descend; but Capricorn that through which they ascend. [102]

Porphyry traces this tradition — of a cave (or cosmos) with two gates — to the Zoroastrian religion, suggesting that this symbolism was transferred to the Greeks in Homer's description of the Cave of Nymphs in the *Odyssey*, in Pythagorean teachings, and in Plato's *Myth of Er*.

Like Porphyry, Origen was also aware that the belief that the soul can ascend through the planetary spheres was not unique to Mithraic mysteries, for Origen writes that "Celsus, too, agreeably to the opinion of Plato, asserts that souls can make their way to and from the earth through the planets…."[103] Origen is referring to Plato's account of the soldier Er, whose soul, released from his wounded body, ascends through the heavens. When Er's soul returns to his body, he awakens and describes his ascent through eight spheres, or "whorls." Er identifies the first and outermost whorl as the sphere of the constellations, inside which the other seven revolve. Er then describes the "widths of the whorls" (i.e., the distances between the planets):

> The first and outermost whorl has the rim broadest, and the seven inner whorls[104] are narrower, in the following proportions — the sixth is next to the first in size, the fourth next to the sixth; then comes the eighth; the seventh is fifth, the fifth is sixth, the third is seventh, last and eighth comes the second. [105]

In column 2 of Table 17, the first and broadest whorl is given the number 1 and each subsequently narrower whorl is given the next consecutive number (2, 3, 4, etc.). Reading from the bottom, up, we have: 4,5,2,6,3,7,8,1. This is close enough to the sequence 4,1,5,2,6,3,7 to give us pause for

102 Porphyry, *The Cave of the Nymphs*.

103 Origen, *Contra Celsum*, Book VI, Chapter XXI, trans. Philip Schaff.

104 Plato numbers the whorls (or spheres) from the outside, in.

105 Plato, *Republic*, Book X, 616e, trans. Benjamin Jowett.

thought, especially since the width of the whorls is not an observable phenomenon — for we have no depth perception when we look into outer space. Instead, as Sir D'Arcy Thompson suggests, "Plato ... was composing a riddle."[106]

Table 17. Widths of the Whorls

Whorl	Width
1st	1
2nd	8
3rd	7
4th	3
5th	6
6th	2
7th	5
8th	4

Knowing what we do about the musical origin of this sequence and its relationship to the rungs on the Mithraic ladder, it is reasonable to imagine that Plato is making an allusion to music. And, in fact, Plato goes on to describe that "on the upper surface of each circle is a siren, who goes round with them, hymning a single tone or note. The eight together form one harmony...."[107]

Yet if Plato is alluding to the musical sequence 4,1,5,2,6,3,7, why is the sequence altered? Plato's original writings do not survive — only copies of copies. Therefore, it is possible that the sequence was unintentionally altered over time. Or perhaps Plato intentionally altered the sequence when committing it to text, for, as we have seen, the sequence played a role in Mithraic initiation and was therefore probably considered privileged knowledge. In any case, we can trace the idea that the spheres are arranged musically as far back as the fifth century BC, for the statement attributed to Pythagoras,

106 http://mathshistory.st-andrews.ac.uk/Extras/Thompson_Plato.html

107 Plato, *Republic*, Book X, 617b, trans. Benjamin Jowett.

quoted earlier, actually reads, in full: "there is geometry in the humming of the strings; there is music in the spacings of the spheres."[108]

What we have seen — with the Mithraic ritual and (perhaps) with Plato — is a description of the *spatial* arrangement of the heavens using the musical sequence 4,1,5,2,6,3,7. It is interesting, therefore, that this sequence was also used in conjunction with a unit of *time*: the seven-day week.

The earliest known reference to a seven-day period is from the reign of the Mesopotamian king Gudea (2144–2124 BC), who built a seven-room temple which he dedicated with a seven-day festival. Further evidence of the importance of a seven-day period is found in the Babylonian Flood Story (1800 BC): "the flood ... swept over the land ... for seven days and seven nights...."[109]

Although a continuous seven-day week is not documented in Mesopotamia,[110] it is known that the Babylonians counted time using periods of seven days — but these periods were adjusted to meet the cycle of the moon: three seven-day periods were followed by an eight- or nine-day period, so that all four "weeks" combined to equal the lunar cycle of 29.5 days.[111] F. Senn attributes the subsequent creation of a continuous seven-day week, independent of the lunar cycle, to the Jews, possibly during the Jewish Captivity in Babylon during the sixth century BC.[112]

The final step in creating the "astrological week," still in use today, was the naming of the seven days after the seven classical planets. The first definite proof of this association is from graffiti at Pompeii, Italy, which was covered by volcanic ash in 79 AD. The graffiti (Figure 70) lists the *dies* (the "days") beginning with *Sat*, an abbreviation of the Latin, *dies Saturni* — "day of Saturn." The other abbreviations of days follow: *Sol*, for *dies Solis* — "day of the sun"; *Lun*, for *dies Luna* — "day of the moon";

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108 *The Houghton Mifflin Dictionary of Biography* (2003): 1250.

109 http://etcsl.orinst.ox.ac.uk/section1/tr174.htm

110 Although CBS 1766 was first interpreted as being an astrological scheme to relate the seven classical planets to the seven days of the week (see footnote 60), there is no proof that the "astrological" week was used in Mesopotamia.

111 T. G. Pinches, "Sabbath (Babylonian)," in *Encyclopedia of Religion and Ethics, ed.* James Hastings (Whitefish, MT: Kessinger Publishing, 2003),20: 889–891.

112 Frank C. Senn, *Christian Liturgy: Catholic and Evangelical* (Minneapolis: Fortress Press, 1997), 323.

Mar for *dies Martis* — "day of Mars"; *Mer*, for *dies Mercurii* — "day of Mercury"; *Iov*, for *dies Iovis* — "day of Jupiter"; and *Ven*, for *dies Veneris* — "day of Venus."

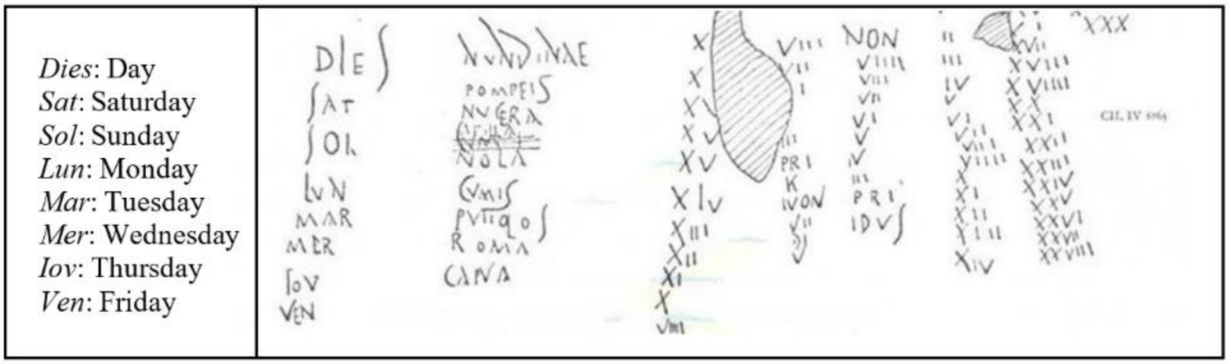

Figure 70. Names of the Days of the Week, Graffiti, Pompeii, Italy (from *Notizie degli Scavi* (1927), 98

Notice that the days aren't named after the planets in the order of their speed, as given by the ancient model of the universe (see Figure 67). Instead, listing the sphere numbers of the planets in their week-day order gives: Sunday (literally, the "first" day in Persian, Hebrew, Greek, etc.): 4; Monday:1; Tuesday:5; Wednesday:2; Thursday:6; Friday:3; Saturday:7. Evidence suggesting that the week was intentionally ordered using the musical sequence 4,1,5,2,6,3,7 is given by Cassius Dio (155–235 AD), who cites two reasons for the week-day order. The first is musical:

> ...if you apply the so-called "principle of the tetrachord" (which is believed to constitute the basis of music) to these stars, by which the whole universe of heaven is divided into regular intervals, in the order in which each of them revolves, and beginning at the outer orbit assigned to Saturn, then omitting the next two named the lord of the fourth, and after this passing over two others reach the seventh, and you then go back and repeat the process with the orbits and their presiding divinities in this same manner, assigning

them to the several days, you will find all the days to be in a kind of musical connection with the arrangement of the heavens.[113]

The "principle of the tetrachord" is a reference to the diatonic scale. For the diatonic scale was described, by the Greeks, as being constructed from two groups of four notes (two "tetrachords"). For example, the scale of C Major is constructed from one tetrachord containing the notes C, D, E and F and a second tetrachord containing the notes G, A, B and C.

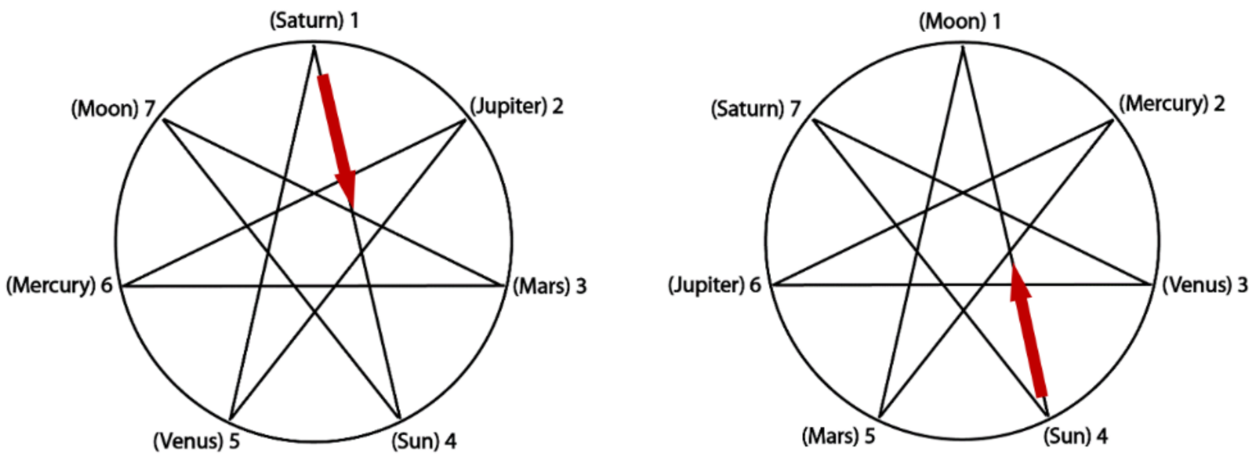

Figure 71: Generating the Week-day Order: (*Left*) Planets Numbered from the Outside, In (re: Plato, Cassius Dio); (*Right*) Planets Numbered from the Inside, Out (re: Ptolemy)

Notice that, like Plato, Dio numbers the planets from the outside, in: Saturn, 1; Jupiter, 2; Mars, 3; the sun, 4; Venus, 5; Mercury 6; the moon, 7. Dio then begins with Saturn (1) and "omitting the next two name[s] the lord of the fourth": the sun (4). Then "passing over two others reach[es] the seventh": the moon (7). The image on the left in Figure 71 shows the planetary names written at the points of a heptagram and numbered as Dio numbers them. Notice that following the diagonals of the heptagram (in the direction shown by the red arrow) illustrates Dio's instructions, generating the sequence 1,4,7,3,6,2,5, which is an inversion, in reverse order, of the sequence 4,1,5,2,6,3,7.

As mentioned earlier, any gearing of the cycles of twelve and seven will generate the sequence

113 Cassius Dio, *Roman History*, Book XXXVII, Chapter 18, trans. Earnest Cary (London: Heinemann, 1914).

4,1,5,2,6,3,7. But this is *not* what Dio is describing. Instead, in the above quotation, Dio describes the gearing of a cycle of seven (the seven planets) with a cycle of four (choose one, skip two, choose one). Why does this generate the sequence 4,1,5,2,6,3,7 (although, in reverse order)?

The image on the right in Figure 71 shows the planetary names written at the points of the heptagram but numbered from the inside, out, as they are in the Ptolemaic model. To generate the week-day order — and the sequence 4,1,5,2,6,3,7 — we start with the sun (4), and follow the red arrow to the moon (1), etc. Notice that, in so doing, we are skipping *three* points each time, thereby generating a cycle of five (choose one, skip *three*, choose one). It becomes apparent, therefore, that Dio's cycle of four and this cycle of five are, in fact, related: they generate the same sequence, but in opposite directions.

But the question still remains: why does the gearing of cycles of five (or four, in the opposite direction) and seven generate the sequence 4,1,5,2,6,3,7? For earlier we saw this sequence to be derived from the gearing of cycles of *twelve* and seven. The answer is that the numbers five and twelve are related in "seven-clock arithmetic." [114] What this means is that, on the "seven-clock" images shown in Figure 71, we can move from point 1 to point 5 *either* by moving five positions clockwise *or* by moving twelve positions clockwise. In other words, in seven-clock arithmetic the gearing of five and seven is the same as the gearing of twelve and seven.

To summarize, Dio's musical reason for the ordering of the days of the week relies on the gearing of cycles of four and seven — the number twelve is not mentioned at all. This is in keeping with the musical tradition of Dio's time, which was a direct inheritance from the early Greeks. For although it is known that the Greeks raised and lowered the pitches of the diatonic scale by one semitone to generate modes, there is no suggestion (as there is with UET VII 74) of their having had knowledge of the cycle of fifths. And if the Greeks were, in fact, unaware of the relationship between the diatonic scale and the circle of fifths, they would not have had knowledge of the irregular heptagram, derived earlier. Instead, the heptagram, if remembered at all, would have been a static shape, with seven equidistant points (Figure 72) — not the rotating model it once was.

114 https://simple.wikipedia.org/wiki/Modular_arithmetic

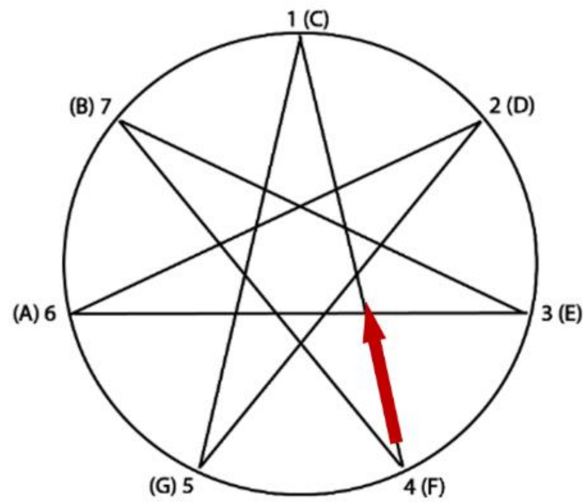

Figure 72: A Static Heptagram Depicting the Diatonic Scale

Figure 72 explains what Dio's "principle of the tetrachord" has to do with the week-day order. For Figure 72 reproduces the image on the right in Figure 71, but instead of writing the names of the planets, we write the notes in the scale of C Major. Following the red arrow generates not only seven consecutive fifths (F, C, G, D, A, E, B) but the sequence 4,1,5,2,6,3,7.

The second explanation given by Dio of the week-day order has to do with the division of the day into 24 hours. According to Dio, each hour of the day was ruled by one of the seven classical planets, in the order generated when listing them according to their speeds, from slowest to fastest. For example, the first hour of Sunday was ruled by its associated planet: the sun. The second hour of Sunday was ruled by the next fastest planet — Venus; the third hour was ruled by the next fastest planet — Mercury; the fourth hour was ruled by the fastest planet — the moon. According to Dio, we now return to the top of the list: to Saturn, the slowest planet — which rules the fifth hour of Sunday. The sixth hour of Sunday is then ruled by Jupiter, and the seventh by Mars. Because there are 24 hours in a day, this sequence will repeat three times (7 × 3 = 21), and three hours will remain. In keeping with the pattern, the first of these remaining hours is ruled by the sun; the second by Venus; and the third by Mercury. Twenty-four hours have now passed, and a new day comes. The first hour of this new day is ruled by the next planet in the sequence: the moon. Consequently, this new day — named for the ruler of its first hour — is called Monday.

Scholars generally dismiss Dio's first, musical explanation. For example, H. Chadwick, in his translation of Origen's *Contra Celsum*, comments on the Mithraic ladder ritual as follows:

> The order of the planets in [the] Mithraic list is not the usual order based on the ancient view of their distances from earth (Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon), but that of the days of the week. Evidently, Celsus mentioned explanations of this order derived from Pythagorean musical theory. Dio Cassius (XXXVII 18–19) also gives two explanations of the planetary week. The second is now regarded as correct. The first is based on the principle of the tetrachord, and is no doubt one of the two mentioned by Celsus.[115]

As we have seen, *any* gearing of the cycles of seven and twelve generates the sequence 4,1,5,2,6,3,7. It is for this reason that listing the planetary ruler of each day generates the week-day order: because there are seven planets and 24 (or 12 × 2) hours. What must be recognized, however, is that division of the day into 24 (or 12 × 2) hours is *arbitrary*. On the other hand, the fact that twelve fifths span seven octaves is a *mathematical truth*: $2/3^{12} \approx 1/2^{7}$. It follows, therefore, that Dio's first, musical reason should not only be given consideration, but precedence.

Further evidence that the week has a musical origin is found in the Christian tradition. For in 321 AD, when the Roman Emperor Constantine decreed the week to be an official unit of time, he also introduced the practice of "octaves," whereby the celebration of certain Church festivals was extended for eight days (Sunday to Sunday, inclusive). This practice accounts for a definition of the word *octave*, still found in most dictionaries: "a period of eight days beginning with the day of a Church festival."[116] Yet, as documented by Church Father Gregory of Nyssa (335–395 AD), the "octave" was not simply a unit of time[117] but a fundamental tenet of the Christian faith:

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115 Origen, *Contra Celsum*, trans. H. Chadwick (Cambridge University Press, 1965), 335, footnote 2.

116 *Oxford Dictionary of English* (3rd ed.), s.v. "octave."

117 Other archaic units of time may have also been based on music. For example, Pentecost, like its Hebrew predecessor *Shavout* (literally "Weeks"), is the day after an observed period of seven times seven weeks ("octaves"). Similarly, a Jubilee

> Thus we accept the law concerning the octave which cleanses and circumcises because once time represented by the number seven comes to a close, the octave succeeds it. This day is called the eighth because it follows the seventh ... and is no longer subject to *numerical succession* [emphasis added]. Another sun makes this day, the true sun which enlightens....[118]

For Nyssa, the week has a spiritual function: to provide a means for the soul to ascend toward God. And not only does the individual undergo this passage; the entire history of the human race is seen as a progression toward the divine — a process that Nyssa calls "akolouthia."

According to Christian tradition, this journey will end when the week-day cycle ceases, for "the nature of time is circumscribed in the week of days."[119] When this seven-day cycle finally comes to a close, time will cease and Christ will return, on what Christians call the "eighth day."

The Greek term that Nyssa uses to describe the transformative journey of the soul — *akolouthia* — translates as "sequence." It is interesting, therefore, that the English word *week* is from Old English *wice*, of Germanic origin; related to Dutch *week* and German *Woche*, from a base probably meaning "sequence, series."[120]

We have seen the sequence 4,1,5,2,6,3,7 used in association with both the measurement of space (the spacings of the spheres) and the measurement of time (the seven-day week). There is, perhaps, a third application of this sequence to measurement.

In ancient times, only three types of measurement were conceived of: that of space (i.e., distance), that of time, and that of weight. In the ancient world, weight was often synonymous with value. For example, the *mina* was, at first, a Mesopotamian unit of weight but became, also (circa 2000 BC), a unit of value equivalent to sixty shekels. (A modern parallel is the *pound*, which is defined as both a unit of weight and a unit of currency.) Yet even before the invention of units of weight, the metals

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year occurs after seven cycles of *shmita*, or sabbatical years (from Hebrew *shabbat*: "Sabbath").

118 Gregory of Nyssa, *On the Sixth Psalm, Concerning the Octave* (J189) (Leiden: E.J. Brill, 1962).

119 Gregory of Nyssa, *On the Sixth Psalm, Concerning the Octave* (J188) (Leiden: E.J. Brill, 1962).

120 *Oxford Dictionary of English*, 3rd ed., s.v. "week."

were seen as having a *relative worth*: a certain amount of gold was worth more than the same amount of silver. But how did this tradition develop? How was it decided that gold is more valuable than silver? To hypothesize an answer, let's look again at the Mithraic ladder.

According to Celsus, the rungs of the Mithraic ladder were made of seven metals, each associated with one of the seven classical planets. This correspondence arose naturally from the fact that there were only seven metals known in the ancient world: gold, sliver, copper, iron, lead, tin and mercury (the supposed chronological order of discovery). Mercury is not mentioned as one of the metals used on the ladder. Instead, Celsus refers to a "mixture of metals" as the material used for the fifth rung. Considering that the manufacture of bronze predated the discovery of mercury, it is probable that Celsus's "mixture of metals" is bronze.

Not all traditions linked the metals and the planets in the same way: some correspondences varied according to time and place. For example, Celsus linked copper with Jupiter, but according to most other sources it was associated with Venus. Nevertheless, three of the correspondences were universal: gold with the sun; silver with the moon; and lead with Saturn.

Table 18. Order of Planets: Apparent Speed

Apparent Speed	Planet	Metal
1	Moon	silver
2	Mercury	iron
3	Venus	tin
4	Sun	gold
5	Mars	bronze
6	Jupiter	copper
7	Saturn	lead

Table 19. Order of the Planets: Mithraic Ladder

Mithraic Ladder	Planet	Metal
4	Sun	gold
1	Moon	silver
5	Mars	bronze
2	Mercury	iron
6	Jupiter	copper
3	Venus	tin
7	Saturn	lead

Table 18 lists the planets in the order of their apparent speeds and gives their associated metals, according to Celsus. Notice that this arrangement of metals is of no recognizable order. However, when we rearrange the planets as they are on the Mithraic ladder — using the sequence 4,1,5,2,6,3,7 (Table 19) — the metals take on a familiar pattern: they become ordered according to their traditional values: gold, the most valued metal, is at the top of the ladder with silver and bronze directly below; lead, the least valued, is at the bottom of the ladder. In other words, the traditional value system of the metals is related to the musical sequence 4,1,5,2,6,3,7.

As mentioned earlier, there is no proof that the Mesopotamians arranged the seven classical planets using the sequence 4,1,5,2,6,3,7. There is however, a theory, proposed in the nineteenth century, that suggests that this planetary order was known in Mesopotamia.

When excavating at Borsippa (ten miles from Babylon), Sir Henry Rawlinson (1810–1895) discovered a cylinder seal describing the ziggurat as "*é.ur.*(*me*)*.imin.an.ki*": "house which gathers the seven (*me*'s) of heaven and underworld."[121] Rawlinson interpreted the "*me*'s" to be "the planets of the seven spheres"[122] and noted that the levels of the ziggurat were surfaced with different colors. Rawlinson

121 A. R. George, *House Most High: The Temples of Ancient Mesopotamia* (Winona Lake, IN: Eisenbrauns, 1993).

122 H. C. Rawlinson, "On the Birs Nimrud, or the Great Temple of Borsippa," *Journal of the Royal Asiatic Society* 18 (1861): 17–18.

then remembered the description by Herodotus (484–425 BC) of the seven concentric battlements of the city of Ecbatana, capital of the Medes Empire (ancient Iran):

> The circles of the walls were, in all, seven. . . . The battlements of the first circle are white, the second black, the third scarlet, the fourth blue, the fifth orange. Thus the battlements of those five circles are painted with colors; but of the last two circles, the one had its battlements coated with silver, the other with gold. [123]

Consequently, Rawlinson formulated a theory suggesting that the Mesopotamians — and, later, the Persians — arranged the seven classical planets according to the weekday order. Flaws in Rawlinson's reasoning, however, led to this theory being rejected by modern scholarship.[124]

However, what we see in Herodotus's description — seven levels, the two highest being silver and gold — is too similar to the Mithraic ladder for Rawlinson's theory to be completely rejected. For we now know that the sequence 4,1,5,2,6,3,7 — which we have seen to generate the value system of the metals, the spacings of the spheres, and the weekday order — was known in Mesopotamia as early as 1800 BC (tablet UET VII 74). Moreover, according to Origen, Celsus traces the origin of this "arrangement" to "musical reasons, quoted by the Persian theology," and Herodotus is describing the ancient Iranian city of Ecbatana.

Another piece of evidence that suggests that the week-day order originated in the ancient Near East comes from the Mandeans, the last surviving Gnostics of antiquity. The Mandeans are native to southern Mesopotamia, and archaeological evidence — Mandean "magical" bowls — attests to their presence in the region from at least as early as the third century AD. Their scriptures, dating from a similar period, are written in the Mandaic language, a variety of Aramaic that shows signs of having

123 Herodotus, *The History of Herodotus*, trans. D. Grene (University of Chicago Press, 1987), 80–81.

124 Nevertheless, this theory has been recently reconsidered by S. Parpola in "Back to Delitzsch and Jeremias: The Relevance of the Pan Babylonian School to the MELAMMU Project" in *MELAMMU Symposia IV*, ed. A. Panainoand and A. Piras. Milan: Università di Bologna & Istituto Italiano per l'Africa e l'Oriente (2004): 237–247.

been influenced, especially in the area of religious and magical terminology, by Akkadian — the language spoken in southern Mesopotamia from the third millennium BC.

According to E. S. Drower, the function and appearance of Mandean priests resemble those of ancient Babylonian priests: Mandean priests, like the Babylonian *baru*, wear white, prepare a sacrament of bread, cleanse themselves in water, and claim Shamish (or Shamash), the sun god, as their special patron. The Mandeans believe that each day is governed by a planet:

> Sunday ... is governed by Shamish; Monday ... is governed by Sin; Tuesday by Nirigh; Wednesday by 'Nbu; and Thursday by Bil (Bel) ... Friday is the day of Libat, and ... Saturday is the day of Kiwan.[125]

The Mandean names for the planetary rulers are drawn, almost completely, from Babylonian lore: the Babylonian sun deity was Shamash; the moon deity, Sin; the deity associated with Mars, Nergal; with Mercury, Nabu; with Jupiter, Marduk (the term Bel comes from Akkadian *bēlu*, signifying "lord" or "master" and was usually used by the Babylonians to denote Marduk); with Venus, Ishtar (who was also called, by the Babylonians, "Dilbat"); and with Saturn, Ninĝirsu. The only Mandean name not connected with Babylonian lore is that associated with Saturn: Kiwan. Yet the Middle Persian name for Saturn, Kēwān, is probably an Akkadian loanword from *kajamānu* "the permanent, steady."[126] To summarize, the Mandeans list the planets, using their Babylonian names, in week-day order — an order derived from the musical sequence 4,1,5,2,6,3,7.

It is interesting, therefore, that during both the Neo-Babylonian period (626–539 BC) and the Seleucid period (312–63 BC), the five planets (and sometimes the sun and the moon) were listed, in Babylonian astronomical and astrological texts, using standardized sequences that have no relationship to observational astronomy. For example, during the Neo-Babylonian period the planets were

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125 Ethel S. Drower, *The Mandaeans of Iraq and Iran: Their Cults, Customs, Magic Legends, and Folklore* (Oxford University: The Clarendon Press, 1973), 74–75.

126 D. N. MacKenzie, "Zoroastrian Astrology in the *Bundahišn*," *Bulletin of the School of Oriental and Afican Studies* 27 (1964): 511–529.

sequentially arranged, in numerous texts, in the order: Jupiter, Venus, Saturn, Mercury, Mars. This arrangement was slightly altered by the Seleucid period to become: Jupiter, Venus, Mercury, Saturn, Mars. These sequences also occur in contexts that are neither astrological nor astronomical. For example, a ritual in the Seleucid period requires that a libation of water for washing hands be offered to "Jupiter, Venus, Mercury, Saturn, Mars, Moon, and Sun, as soon as they appear."[127] What is important, here, is that textual evidence shows that the practice of listing the planets sequentially, but in non-astronomical order, was already present in Babylon as early as 600 BC. This suggests, once again, the strong possibility of a Near Eastern origin for the use of the musical sequence 4,1,5,2,6,3,7 in creating the astrological week — for, as previously mentioned, by this date, a period of seven days had already been in use in Mesopotamia for over a millennium.

F. Rochberg-Halton proposes that both the Neo-Babylonian and Seleucid sequences are derived from the fact that certain planets were considered, by the Babylonians, to be benefic, while others were thought to be malefic. For in both sequences, the planets that were considered benefic (Jupiter and Venus) are listed first.[128]

For the Mandeans, too, the planets are both beneficent and maleficent. The sun, for example, is regarded as overwhelmingly beneficent, as shown by the following statements — given in the 1930s to E. S. Drower by Mandean informants — that describe the beneficent character of the god Habshaba (literally "First-Day-of-the-Week"), an aspect of Shamish, the sun god:

> Sunday, which is governed by Shamish, is also associated with the personified Habshaba, First-Day-of-the-Week … [Mandean informants say that:] "The gate of the World of Light is ajar on this day and Habshaba takes the souls by means of electricity into the midst of the world of light." … "Habshaba descends into Mataratha (Purgatories) on

127 TCL 6 41:23f, trans. A. Sachs in ANET 2: 338.

128 Francesca Rochberg-Halton, "Benefic and Malefic Planets in Babylonian Astrology," *Scientific Humanist: Studies in Memory of Abraham Sachs* (Philadelphia: University of Pennsylvania, 1988): 319–324.

> Sunday, returning with seven Mandaean souls to the world of light." ... "The revolving wheels of light whirl more swiftly on this day, thus assisting the souls in their ascent."[129]

Yet all seven classical planets are also thought, by the Mandeans, to be able to exert negative influences. M. Bloomfield, when discussing this Mandean belief, suggests that the Christian concept of seven cardinal sins came directly from the pre-Christian belief that negative influences are exerted by the planets on the soul. Bloomfield points out that Reitzenstein[130] (and Bousset[131]) traces the origin of this belief, and the related concept of the "soul journey," to Persia.

> Many Hellenistic sects believed that the soul, after death, had to journey through the seven zones of heaven, while the ariel spirits ... attempted to hinder its passage.... Persia may have been the ultimate home of the Soul Journey. Reitzenstein suggests that a representation of the Soul Journey (which he calls the Soul Drama) is the basic concept of the Persian folk-religion which passed to the west through Babylon into various religions, picking up certain Chaldean beliefs on the way.[132]

M. Boyce identifies the elements of the Persian folk-religion as originating with the proto-Indo-Iranians who "forged a religious tradition of immense strength, so that to this day elements from it are preserved by their descendants, the Brahmans of India and the Zoroastrians of Iran."[133]

Specifically, Boyce describes the people who followed this "old religion" as having had a "belief in life after death for the individual, and according to its earliest form the disembodied spirit, the "*urvan*,"

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129 Drower, *The Mandaeans of Iraq and Iran*, 74–75.

130 R. Reitzenstein, *Das iranische Erlösungsmysterium, Religionsgeschichtliche Untersuchungen* (Bonn: Marcus et Weber, 1921).

131 W. Bousset, *Himmelsreise der Seele* (*Sonderausgabe*) (Darmstadt: Wissenschaftliche Buchgesellschaft, 1960).

132 M. W. Bloomfield, "The Origin of the Concept of the Seven Cardinal Sins," *The Harvard Theological Review* 34 (no. 2) (1941): 121–128.

133 Mary Boyce, *Zoroastrians: Their Religious Beliefs and Practices*. Library of Religious Beliefs and Practices. (London: Routledge/Kegan Paul, 2001), 2.

lingered on earth for three days before departing downward to a subterranean kingdom of the dead…."[134] In other words, the proto-Indo-Iranians believed that the souls the dead *descended* to the underworld. Yet, as Boyce explains, shortly before the proto-Indo-Iranians drifted apart (1800–1600 BC) to become identifiable by speech as two distinct peoples — the Indians and the Iranians — there developed a belief that the soul can ascend to a celestial paradise:

> It seems probable that it was just before the Indians and Iranians separated that they conceived a new hope concerning the hereafter. This was that some at least among them —princes and warriors and the priests who served the gods — might escape the dreaded fate of an eternally joyless existence, and that their souls might mount upward at death to join the gods in sunlit Paradise….[135]

To summarize, Boyce, Reitzenstein, and Bousset trace the origin of the concept of the "soul journey" to Persia. Furthermore, as we have seen, Celsus associates the arrangement of the planets on the Mithraic ladder with "musical reasons … quoted by the Persian theology." Moreover, as previously mentioned, the musical system of Mesopotamia is thought to have been in use in Elam (later, Persia) as early as 1800 BC. It is suggested here, therefore, that the association of the ascension of the soul with music originated in Persia, probably in the early first millennium BC.[136]

In the Mithraic ladder ritual, the metaphor of music is used to describe not only "cosmic" analogies — the spacings of the spheres, the week-day order, and the value system of the metals — but the human postmortem experience. It is interesting, therefore, that the sequence 4,1,5,2,6,3,7 may also have played a role in alchemical tradition. For although alchemy is popularly known as being a process undertaken to transform base metals into gold, many scholars believe that this was simply a metaphor for the transformation of the soul.

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134 Boyce, *Zoroastrians*, 12.

135 Boyce, *Zoroastrians*, 14.

136 According to Herodotus, Ecbatana was chosen as the Medes' capital in the late eighth century BC (*The History of Herodotus*, trans. D. Grene [University of Chicago Press, 1987], 80–81.)

To identify a possible relationship between alchemy and the soul journey, recall that when derived from the spiral of fifths, the sequence 4,1,5,2,6,3,7 repeats indefinitely — because there are an infinite number of both octaves and fifths. When we write the sequence as repetitive, the number 7 follows the number 4: 4,1,5,2,6,3,7 ... 4,1,5,2,6,3,7.... According to the Mithraic ladder, lead is associated with Saturn (7), and gold with the sun (4). Therefore, moving from 7 to 4 (i.e., moving from one cycle of the sequence to the next) is the equivalent of moving from lead to gold. It is interesting, therefore, that Eliade, in discussing rituals of ascent that incorporate the number seven, says that:

> A ladder with seven rungs was also preserved in alchemical tradition. A codex [in the Royal Library at Modena] represents alchemical initiation by a seven-runged ladder up which climb blindfolded men; on the seventh rung stands a man with the blindfold removed from his eyes, facing a closed door. [137]

To summarize, the sequence 4,1,5,2,6,3,7 was used in conjunction with the measurement of space, time and value/quality (both the material value of the metals and the spiritual condition of the soul). These applications of the sequence are *artificial human constructs*. We can deduce from this that the sequence 4,1,5,2,6,3,7 was not only known in the ancient world, but given great importance. This supports the earlier hypothesis that the diatonic scale is a human construct, rather than a natural phenomenon: for the artificial use of the sequence to describe the spacings of the spheres, the seven-day week, the value system of the metals, and the soul journey implies that the diatonic scale is, in fact, an earlier but nevertheless *intentional* application of the same sequence.

The sequence 4,1,5,2,6,3,7 is derived from the mathematics of music. It is interesting, therefore, that, according to Aristotle (384–322 BC), there existed a Pythagorean tradition whereby "the whole of nature" and the greater cosmos was believed to be modelled on the numbers of music:

> the ... Pythagoreans ... saw that the modifications and the ratios of the musical scales were expressible in numbers; — since, then, all other things seemed in their whole

137 Eliade, *Shamanism*, 490.

> nature to be modelled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and *the whole heaven to be a musical scale and a number* [emphasis added]. And all the properties of numbers and scales which they could show to agree with the attributes and parts and the whole *arrangement of the heavens* [emphasis added], they collected and fitted into their scheme; and if there was a gap anywhere, they readily made additions so as to make their whole theory coherent.[138]

Aristotle describes the numbers of music as corresponding with the "arrangement of the heavens." We have seen this phrase — "arrangement of the heavens" — used before: by Cassius Dio, when describing the musical reason that accounts for the weekday order. Moreover, Celsus examines "the reason of the stars being arranged in this order ... [and gives] ... musical reasons."

Aristotle's account is the first historical mention of a scheme linking mathematics, music, and the cosmos — a tradition that came to be referred to, in the West, as "the music of the spheres." Were the Pythagoreans the first to conceive of this idea? Probably not, for remember, according to Iamblichus, Pythagoras studied both mathematics and music with the Persian Magi, in Babylon.

The Magi were Zoroastrian priests — the word magic being derived from the name Magi. Their homeland was Media, a region of northwestern Iran. (It was the Median city of Ecbatana that Herodotus described as having seven colored battlements.) In 612 BC the Medes formed an alliance with Babylon to overthrow the Neo-Assyrian empire. From this time onward, continuing in the Achaemenid period (500–330 BC), there was a strong Mesopotamian influence on Persian culture, specifically in the areas of art and religion — an exchange that was stimulated by the presence of the Magi in Babylon.

The Magi traveled widely, affecting religious beliefs in the regions they visited. It is probable, therefore, that they influenced the Roman Mithraic Mysteries,[139] which, as we have seen, encoded the sequence 4,1,5,2,6,3,7 in the ladder initiation ritual.

In summary, we have seen that, in the West, the tradition of the music of the spheres — which

138 Aristotle, *Metaphysics*, Book I, Chapter 5 (985b23–986a6), trans. W. D. Ross (Buffalo: Prometheus Books, 1991).

139 Franz Cumont, *The Mysteries of Mithra*, trans. T. J. McCormack (Chicago: Open Court, 1903), 24.

envisioned the architecture of the cosmos as being related to the numbers of music — created a complex system of analogies.[140] It is interesting, therefore, that in China, beginning in the Warring States period (475–221 BC), the idea of *ganying* (感應)or "correlative resonance" appears in texts to denote a cosmological principle that is described, analogically, using the principles of music. As we'll now see, this concept of correlative resonance was so important that it played a vital role not only in Chinese philosophical tradition, but in all aspects of Chinese life.

140 The word *analogy* comes from the Greek *analogos*, which is made up of two words: *ana* ("upon") and *logos* ("reason," "word," "ratio"). Could the concept of analogy have a mathematical origin: was an analogy originally a comparison built upon (*ana*) number (*ratio*)? For this is how the sequence 4,1,5,2,6,3,7 is used: as a numerical link by which a sequence from the proportions (*analogia*) of music is re-applied to create non-musical correspondences.

16. MUSICO-COSMOLOGY IN ANCIENT CHINA

The ancient Chinese conceived of the universe as an interconnected whole, a tradition that is often described as "correlative cosmology."[141] The functioning of the cosmos was thought to depend, primarily, on the interplay of two rhythms or cycles: the "five phases" (*wǔxíng*: 五行) and *yin-yang* (陰陽). It was believed that patterns of change are a result of the cyclical interactions of the five phases and the vital forces (*qi*: 氣) of *yin* and *yang*. This relationship was thought to manifest itself in both the macrocosm and microcosm: in the celestial patterns; in the organization of government; and in the internal structure of the human body. Consequently, the five phases theory, coupled with *yin-yang*, played a fundamental role in all aspects of traditional Chinese life.

The English translation of *wǔxíng* (五行) is *wǔ* (五: "five") and *xíng* (行: "moving"). In the Chinese language, a planet is called a "moving star" (行星: *xíngxīng*) and, originally, *wǔxíng* referred to the five visible planets: Mercury, Venus, Mars, Jupiter, and Saturn. Similarly, the terms *yin* and *yang* correspond, traditionally, to the moon and the sun, respectively.

In the ancient world, this division of the seven classical planets into a group of five and a group of two was not exclusively a Chinese concept:

> In antiquity, only five planets (Mercury, Venus, Mars, Jupiter, Saturn) were known…; in many early traditions, and also in some Iranian contexts, the two so-called "luminaries," the Sun and the Moon, were added to their number. For this reason, some sources mention both the "five" and the "seven" planets.[142]

As previously mentioned, in the Near East, as early as the Neo-Babylonian period (626–539 BC), the five visible planets were listed sequentially, in an order that was not astronomical. It is interesting, therefore, that, three centuries later, ancient Chinese texts list the five visible planets in sequences that are also of non-astronomical order. And just as, in the West, each planet became associated with a set

141 See, for example: https://plato.stanford.edu/entries/chinese-metaphysics/

142 A. Panaino, "Planets," *Encyclopædia Iranica*, online edition (2016): https://iranicaonline.org/articles/planets.

of non-linear correspondences (for example, Saturn: lead; Saturday, etc.), so was the Chinese system based on similar seemingly irrational[143] correlations (Table 20):

Table 20. Five-Phase Theory Correspondences

Planets	**Jupiter**	**Mars**	**Saturn**	**Venus**	**Mercury**
Elements	wood	fire	earth	metal	water
Seasons	spring	summer	high summer	autumn	winter
Directions	east	south	central	west	north
Tastes	sour	bitter	sweet	pungent	salty
Emotions	anger	joy	sympathy	grief	fear
Colours	blue-green	red	yellow	white	black
Tones	*jué*	*zhǐ*	*gōng*	*shāng*	*yǔ*
Heavenly Stems	*jia*, *yi*	*bing*, *ding*	*wu*, *ji*	*geng*, *xin*	*ren*, *gui*

(Information selected from Table 75, Section 73.1, Wilkinson's *Chinese History: A Manual*, 2018)

> The universe, according to this [the ancient Chinese] view, is a harmoniously functioning organism consisting of multitudinous objects, qualities and forces which … are integrated into coherent patterns by being subsumed under one or another of many numerical categories. (The best known such category, of course, is that in sets of fives, such as the five elements, five directions, five colors, etc.) *Among items belonging to a common category, a particular affinity exists between those having the same relative position within their respective sequences* [emphasis added]. For example, the property common to such diverse items as fire, summer, south, bitter taste, burning smell, heat, the planet Mars, feathered creatures, beans, the hearth sacrifice, the lungs, the tongue,

143 The modern Western mind is taught that rational, causal thought is the only valid form. As a response to this, C. J. Jung refers to the correlations in the *I Ching* not as chance, but as "acausal." (C. G. Jung, foreword to the *I Ching or Book of Changes*, trans. R. Wilhelm, Princeton University Press, 1950: xxiv). Similarly, W. A. Callahan refers to Daoist thought not as irrational but "arational." (W. A. Callahan, "Discourse and Perspective in Daoism: A Linguistic Interpretation of Ziran," *Philosophy East and West* 39, 1989: 171).

> joy, and many more,[144] is that each of them is number two within its particular sequence of five. Affinities of this kind should be thought of as functioning … along lines of spontaneous response (the response of one stringed instrument to another the same in pitch)….[145]

According to D. Bodde, quoted above, affinities "should be thought of as functioning … along lines of spontaneous response (the response of one stringed instrument to another the same in pitch)." This statement is based on the use, in ancient texts, of the analogy of *ganying* (感應), or "correlative resonance," to explain the affinity between things of the same category. An example is the following passage from the *Lǚshì chūnqiū* (239 BC):

> Things belonging to the same category naturally attract each other; things that share the same ethers [*qi* 氣] naturally join together; and notes that are comparable naturally resonate [應] to one another. Strike the note *gōng* on one instrument and other strings tuned to the *gōng* note will vibrate [應]; strike the note *jué* and other strings tuned to the *jué* note will vibrate [應]. [146]

> 類同相召，氣同則合，聲比則應.鼓宮而宮動，鼓角而角動.

In the above quote, sympathetic resonance or *ganying* (for example, between multiple *gōng* notes or multiple *jué* notes) is used as an analogy to describe the force that underlies the five-phase correspondences. *Ganying* is a process of interaction that transcends time and space — and rational,

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144 A correspondence not mentioned by Bodde is the "Heavenly Stems" — a list of ten names, first used circa 1250 BC, to name the days in a ten-day "week." As shown in Table 20, a pair of Heavenly Stems are linked to each element in the five-phase system.

145 Derk Bodde, "The Chinese Cosmic Magic Known as Watching for the Ethers," *Essays on Chinese Civilization* (Princeton University Press, 1982), 351–352.

146 *Lǚshì chūnqiū* (春秋), Book 13, Ch. Zhao Lei (召類), trans. Knoblock and Riegel, in *The Annals of Lü Buwei*,283.

linear causality. Through the mechanism of resonance, an event in one location produces an effect in another location, and the medium by which this dynamic influence is exchanged is *qi*. For "things belonging to the same category" (i.e., having the same relative position within their respective sequences) "naturally attract each other."

The earliest textual reference to the five phases is found in the third century BC:

> "The Five [Elements] are: water, fire, wood, metal and earth. Water pours down; fire blazes and rises; wood is either crooked or straight; metal does as it's commanded; earth sprouts crops. What soaks becomes salty; what burns becomes bitter; what is crooked or straight becomes sour; what is hard but melts becomes acrid; what is sown and reaped is delicious!" [147]

> 五行：一曰水，二曰火，三曰木，四曰金，五曰土. 水曰润下，火曰炎上，木曰曲直，金曰从革，土爰稼穑.润下作咸，炎上作苦，曲直作酸，从革作辛，稼穑作.

In the preceding quotation, the five elements are given in the order: water, fire, wood, metal, earth (or, to list the corresponding planets: Mercury, Mars, Jupiter, Venus, Saturn) — an order that is different from that given in Table 20. Yet both were traditionally accepted orders. In fact, numerous variations of the sequence were used: according to W. Eberhard,[148] sixteen variations of the sequence are found in pre-Han and Han texts. Eventually, two sequences — "mutually generating" (wood, fire, earth, metal, water: the order given in Table 20) and "mutually overcoming" (wood, earth, water, fire, metal) — became more or less standard and are still used today in the practice of Chinese medicine.

V. Rubin[149] proposes that the five-phase theory was originally independent from the idea of *yin-yang*, and constituted a spatial model — as opposed to the temporal model of the cycling activity of

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147 *Shūjīng* (書經), Ch. *Hóng fàn* (洪範), trans. M. Palmer, J. Ramsay, V. Finlay, in *The Most Venerable Book* (*Shang Shu*) (Penguin Classics, 2014), 152.

148 W. Eberhard, "Beiträge zur Kosmologischen Spekulation Chinas in der Han-Zeit," *Baessler-Archiv* 16 (1) (1933).

149 V. Rubin, "The Concepts of Wu-Hsing and Yin-Yang," *Journal of Chinese Philosophy* 9 (1982): 131–157.

yin-yang. In fact, a spatial model is one way in which the five-element theory is still depicted today: the five elements are written at the points of a pentagram (Figure 73). When following the outer circle of this model, the sequence of mutual generation is derived; following the diagonals of the pentagram gives the order of mutual overcoming.

Early Chinese schools of philosophy used the concept of *ganying* to account for relationships of resonance within the cosmos: in Daoism, *ganying* represented the universal resonance between things in the natural world; in Confucianism, there arose the concept of *tian-ren ganying* (天人感應) — described as "ethical resonance" between heaven and the human realm; later, in Chinese Buddhism, *ganying* came to signify "prayers being heard."

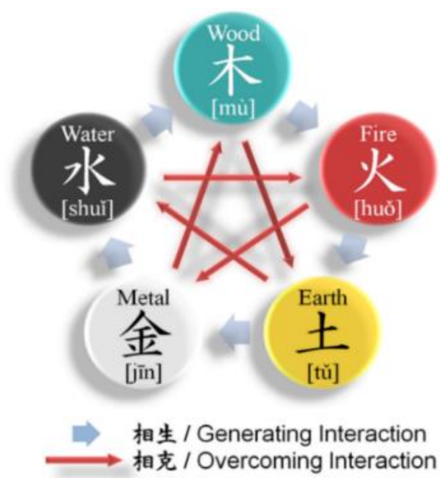

Figure 73: Five-Element Theory: A Spatial Model (Source: Wikipedia)

C. Le Blanc gives the following synopsis of the concept of *ganying* (which he refers to as "*kan-ying*"), as it is described in the *Huainanzi* (淮南子) — a text that blends Daoist and Confucianist thought, written during the second century BC:

> *Kan-ying* may be defined ... as the power of things to affect and to be affected in such a way as to bring about harmony. This power is based on the persistent affinity and attraction of things that were originally one, but that became scattered when the world began. Through the True Man *kan-ying* recreates the original unity. As a dynamic

> pattern *kan-ying* expresses the full cycle of cosmological, social and psychological integration. Its natural and universal character makes it binding for the cosmos as a whole and also for each and every one of the Ten Thousand Things issuing from Tao. In Chapter Six the foregoing pattern is applied mainly to the realm of human society and, more specifically, to the relations between the perfect ruler — the True Man — and the people. The argument there propounded, that these relations should be based on non-action (*wu-wei*) understood as resonance (*kan-ying*), draws its ultimate strength from the cosmological scheme outlined above. We may thus conclude that *kan-ying* not only forms a logically coherent and philosophically meaningful idea but also provides the focal point around which the Huai-nan Tzu cosmology is structured. [150]

In the above quotation, Le Blanc describes the Confucianist idea of ethical resonance, whereby the perfect ruler, the True Man, acts according to the principle of *ganying* — without force yet responding to stimulus in a spontaneous manner, in keeping with the principles of harmonic resonance. The level of the ruler's embodiment of *ganying* is of cosmic importance, for the quality of his moral character is directly proportionate to the prosperity of the realm.

By the second half of the first millennium AD, the concept of *ganying* became central to Chinese Buddhism. For example, Chapter Five of the *Dasheng Xuanlun*, written by Zhizang (458–522 AD) explains the importance of *ganying* ("stimulus-response") as follows:

> Stimulus-response is the great tenet of the buddha-dharma, the essential teaching of the many sutras. To "stimulate" means to bring or summon forth, and to "respond" means to go forth and meet in welcome. As all sentient beings possess [the seeds of] goodness, they may induce the Buddhas to descend and take shape in front of them, and [the Buddhas] will meet them in welcome. The principle [is such that they] neither deviate nor overshoot [the mark]. This is called stimulus and response. The common

150 C. Le Blanc, *Huai-nan Tzu: Philosophical Synthesis in Early Han Thought: The Idea of Resonance* (*Kan-Ying*), with a translation and analysis of Chapter Six (Hong Kong University Press, 1985), 209.

person stimulates but does not respond; the Buddhas respond but do not stimulate; and bodhisattvas both respond and stimulate.[151]

感应者乃是佛法之大宗.众经之纲要.言感者牵召义.应者赴接义.众生有善致彼佛前.垂形赴接.理无乖越.谓之感应.凡夫感而不应.诸佛应而不感.菩萨亦应亦感.

To summarize, the principle of *ganying* was identified as the force responsible for establishing correlations linking the cosmic realm, the human realm, and the natural world. As mentioned earlier, these correlations were demonstrated by creating categories in which items were listed sequentially and then cross-related, to illustrate affinities.

As described in the previously quoted comment from Bodde, the most common category was that of sets of fives. Yet there were other categories — the next most important being the category based on the number twelve.

Like the Babylonians, the ancient Chinese divided the ecliptic into twelve constellations — or *ci* (次). The *ci* were originally conceived of as the "stations" of Jupiter (which takes approximately twelve years to circle the sun, and consequently appears, from earth, to occupy each *ci* for a period of approximately one year). In keeping with the practice of correlational thinking, other categories of twelve were associated.

As mentioned earlier (see footnote 144), the "Heavenly Stems" were, originally, the names of the days in a ten-day week. Yet the ancient Chinese time-keeping system also used twelve "Earthly Branches." (Eventually, these two groups were fused together in the Stems-and-Branches (干支) system, to form a sixty-count cycle that was used, as early as 1200 BC, to create a cycle of sixty days — as evidenced by textual references on the Shang oracle bones. By the Han dynasty [202–220 AD], this sixty-day period was being associated with the "sixty different sequences or scales, known ... as the sixty pitch-pipes"[152] [i.e., the 5 × 12 = 60 pentatonic modes: see footnote 58]. Here, then, we see a correlation made between

151 Zhizang, *Dasheng Xuanlun* (大乘玄論), Chapter 5, "Chinese Buddhism and the Cosmology of Sympathetic Resonance," trans. R. H. Sharf in *Coming to Terms with Chinese Buddhism* (Honolulu: University of Hawai`i Press, 2002), 121.

152 Bodde, "Watching for the Ethers," 354.

a unit of time and sequences found in music that brings to mind the use of the sequence 4,1,5,2,6,3,7, in the West, in ordering the days of the seven-day week.) The twelve Earthly Branches were directly correlated with the twelve *ci*, and other twelvefold categories were added.

Table 21. Categories in the Ancient Chinese Correspondence System Based on Twelve

Earthly Branches	*zǐ*	*chǒu*	*yín*	*mǎo*	*chén*	*sì*	*wǔ*	*wèi*	*shēn*	*yǒu*	*xū*	*hài*
Lunar Months	11	12	1	2	3	4	5	6	7	8	9	10
Zodiac Animals	Rat	Ox	Tiger	Rabbit	Dragon	Snake	Horse	Goat	Monkey	Rooster	Dog	Pig
Pitch Pipes	*Huáng Zhōng*	*Dà Lǚ*	*Tài Cù*	*Jiá Zhōng*	*Gū Xiǎn*	*Zhòng Lǚ*	*Ruí Bīn*	*Lín Zhōng*	*Yí Zé*	*Nán Lǚ*	*Wú Yì*	*Yìng Zhōng*
Directions	0° (N)	30°	60°	90° (E)	120°	150°	180° (S)	210°	240°	270° (W)	300°	330°
Dual Hour	23:00	1:00	3:00	5:00	7:00	9:00	11:00	13:00	15:00	17:00	19:00	21:00

For example, in the *Liji* (禮記) (third century BC), twelve lunar months are listed and correlated with the twelve *lǜ*, in their chromatic order (see Table 2): the longest pipe, *Huáng Zhōng*, is associated with the second month of winter (which contains the winter solstice), and so on through the cycle of the seasons — winter, spring, summer, autumn — till the shortest pipe, *Yìng Zhōng*, is associated with the first month of winter. Other categories in the twelvefold system include the twelve animals of the Chinese "zodiac"[153]; a directional system used primarily by ancient Chinese mariners and astronomers based on a twelve-part (30°) division of the circle of the horizon; and a division of the day into twelve "double-hours" (Table 21).

By the third century BC, this twelvefold system became associated with a standardized system of weights and measures — and the element that provided the correlative basis was music:

153 The term "zodiac" is a misnomer, for there are several differences between the Western (Babylonian) zodiac and the Chinese system. Two fundamental differences stand out: firstly, the animals of the Chinese zodiac are not associated with the constellations that make up the ecliptic; secondly, the Chinese twelve-part cycle corresponds to years, not months.

> While other early civilizations concerned themselves with linear measure, capacity, and weight in formulating their metrological systems, the Chinese were apparently unique in including pitch-measure (*lǜ*), and that not merely on par with, but as the basis of, the other three. [154]

This metrological system is described the *Hanshu* (漢書), written in the first century AD:

> The basis of linear measure is the length of the *Huáng Zhōng* (pitch-pipe). Using grains of black millet, the length of *Huáng Zhōng* is ninety *fen* (one *fen* being equal to the width of a grain of millet) ... twelve hundred (grains) fill its tube.... The contents of one tube weigh twelve *chu*.[155]

> 本起黃鐘之長.以子穀秬黍中者，一黍之廣，度之九十分，黃鐘之長……一龠容千二百黍，重十二銖.

In order to establish the absolute units described in the *Hanshu*, it was necessary to determine the exact length (and therefore, the exact pitch) of the *Huáng Zhōng*. The ancient method for determining the pitch of the *Huáng Zhōng*, and consequently all twelve *lǜ*, was related to the cosmic, cyclic flow of *qi*. This is documented in the *Hanshu*: "the *qi* of heaven and earth combine and produce wind. The windy *qi* of heaven and earth correct the twelve pitch fixations. [天地之氣合以生風；天地之風氣正，十二律定.]" [156]

A thousand years after the *Hanshu* was written, the practice of determining the pitch of the *lǜ* using *qi* was still in use, as described by the philosopher Tshai Yuan-Ting (1135–1198 AD):

154 Needham and Robinson, *Science and Civilisation*, 199.

155 *Hanshu*, Chapter 21 (律曆志), trans. Needham and Robinson, in *Science and Civilisation*, 201.

156 *Hanshu*, Chapter 21 (律曆志), trans. Needham and Robinson, in *Science and Civilisation*, 187.

> The (pitch-pipes) are blown in order to examine their tones, and set forth (in the ground) in order to observe (the coming of) the *chhi* [*qi*]. Both (these techniques) seek to (determine the correctness of the) Huang-chung [*Huáng Zhōng*] tube by testing whether its tone is high or low [i.e., clear or muddy], and whether its *chhi* (arrives) early or late. Such were the ideas of the ancients concerning the making (of the pitch-pipes).... If one desires to find the middle (i.e., the correct) tone and *chhi* without having anything available as a standard, the best thing to do is to cut several bamboos for determining the right Huang-chung length, making some shorter and some longer.... If this having been done one blows them one by one, the middle (i.e., the correct) tone will be obtained, and if one sets them more or less deeply (in the ground), the middle (i.e., the correct) *chhi* may be verified. When its tone is harmonious and its *chhi* responds, the Huang-chung is really a Huang-chung indeed. And once it is really so, then (from it) may be obtained the (other) eleven pitch-pipes, as well as the measures of length, capacity and weight. Later generations, not knowing how to go about this, have sought (to construct accurate pitch pipes) only by measuring with the foot-rule.[157]

> 吹以考聲，列以候氣.皆以聲之清濁，氣之先後，求黃鐘者也.是古人制作之意也…今欲求聲氣之中而莫適為准則，莫若且多截竹以擬黃鐘之管.或極其短，或極其長…如是而更迭以吹，則中聲可得.淺深以列，則中氣可驗.苟聲和氣應，則黃鐘之為黃鐘者信矣.黃鐘者信，則十一律與度量衡權者得矣.後世不知出此，而唯尺之求.

What Tshai Yuan-Ting is describing here is a practice known as the "watching for the ethers" or the "blowing of the ashes." This practice relied on the belief that although *qi* is invisible, its arrival could be detected by the twelve *lǜ*. The procedure was as follows: the *lǜ* were buried upright in the ground, in circular arrangement, with each pipe extending a few inches above the soil. This was done within a well-sealed chamber, with an earthen floor. The *lǜ* were then filled with ashes. The theory was that each

157 Tshai Yuan-Ting, *Lü Lü Hsin Shu*, Ch. 2, sect. 1, trans. Bodde, in Needham and Robinson, *Science and Civilisation*, 187.

month the *yin* or *yang qi* of that month would rise, causing the ashes to be blown from the upper end of only one pipe — the pipe corresponding to that month. The earliest detailed description of this process is credited to Ts'ai Yung (132–192 AD) and is recorded in the *Hòu Hànshū* (後漢書), the "Book of the Later Han" (fifth century AD):

> As to the procedure for watching the ethers [*qi* 氣]: a triple-walled chamber is prepared, the doors of which bar it off (from the outside world).... Stands are made out of wood, one for each pitch-pipe, which extend deep down within (the ground) and high up outside (the ground). The pitch-pipes, in accordance with their compass points, are mounted upon these. Ashes from the pith of reeds are stuffed into the inside (of each pitch-pipe), and in accordance with the calendar, a watch is kept upon them. Whenever one (of the pitch-pipes) is reached by the ether (of its corresponding month), its ashes move.[158]
>
> 候气之法，为室三重，户闭，涂衅必周，密布缇缦.室中以木为案，每律各一，内庳外高，从其方位，加律其上，以葭莩灰抑其内端，案历而候之.气至者灰去.

We see, therefore, that the practice of "watching for the ethers" had a correlative function, for not only was *qi* used to determine the pitch of the *Huáng Zhōng*, the pitch pipes were used to indicate the arrival of the *qi*. In other words, music and cosmology were fundamentally linked.

No wonder, then, that in ancient times, those people thought most able to perceive the *tao* of heaven were astronomers and musicians — usually blind musicians. For example, according to a story in the *Guo Yu* (fifth century BC), when Shan Hsiang-kung predicted that there would be unrest in the state of Chin, the Duke of Lu said to him:

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158 *Hou Han shu* (后汉书), Rhythm and the Calendar (律历上), Section One, trans. Bodde, "The Chinese Cosmic Magic Known as Watching for the Ethers," *Essays on Chinese Civilization* (Princeton University Press, 1982), 356.

> "Now you, my lord, say that there will be disturbances. May I presume to ask whether it is the *tao* of heaven, or is it because of men?" He replied, "I am no blind (musician) or historian-astronomer. How could I know the *tao* of heaven?"[159]
>
> 魯侯曰："寡人懼不免於晉，今君曰'將有亂'，敢問天道乎，抑人故也？"對曰："吾非 瞽、史，焉知天道？

In fact, the archetypal blind musician was not only believed to have knowledge of the *tao* of heaven, but to have originally determined the proportions of the *shí'èr lǜ*:

> "The divine blind men of old examined the true notes and measured them in order to establish regulations."[160]
>
> 古之神瞽考中声而量之以制，度律均锺.

It is important to note, that the *lü* were, originally, more than standardized pitches: they were considered magical instruments. For example, as Needham describes, there are "early references to the shaman-musician piping off his own *chhi* [*qi*] through bamboo tubes in an attempt to alter the process of Nature — of heaven's *chhi* [*qi*] — by sympathetic magic." [161] Needham is referring, here, to the *wu* — the traditional name of the ancient Chinese shaman.

Eliade describes the initiation ceremony of the *wu* as follows: "The ceremony consists ... in mounting the *to t'ui* (the "sword-ladder") ... usually the ladder is made of twelve swords."[162] The fact that the *wu* shaman climbs a ladder of twelve steps suggests a musical association for not only are there

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159 *Chou Yü* III, Section 1, trans. J. Pinckney Hart, Jr., "The Philosophy of the Chou Yü" (1973), 357.

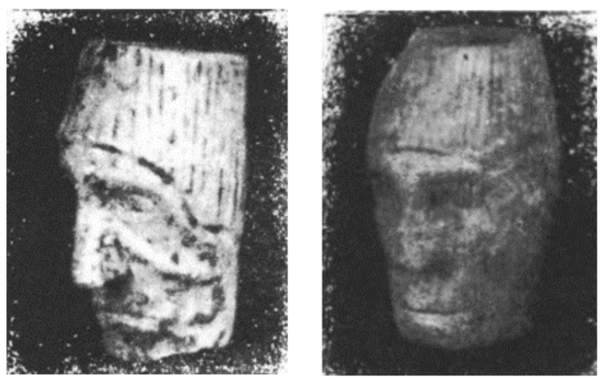
160 *Chou Yü* III, Section 7, trans. J. Pinckney Hart, Jr., in "The Philosophy of the Chou Yü," (1973), 396.

161 Needham and Robinson, *Science and Civilisation*, 134–135.

162 Eliade, *Shamanism*, 455.

twelve *lǜ*, the term *lǜ*, which is generally translated as "pitch-pipe," originally meant "regular steps."[163] As we have seen, the twelve *lǜ* are identical to the twelve pitches generated by the re-tuning cycle on UET VII 74. It is interesting, therefore, that not only is the *wu* ascension rite reminiscent of the Mithraic ladder ritual, the word *wu* may have linguistic ties to the Near East.

In 1980, archeologists excavating a Western Chou palace in Shensi discovered two small carved heads (Figure 74), dated circa 800 BC. The consensus is that the individuals depicted are Europoid.[164] Carved on the top of head T45:6 is the symbol ☩ (Figure 75). This symbol also appears on pottery dating to 5500 BC from Tell Halaf, a site in Upper Mesopotamia. V. Mair points out that this symbol was, in the West, "the symbol of the magician through the Middle Ages until the present time,"[165] and was referred to as the Cross Potent. Mair notes that "its shape is identical to the earliest form of the Chinese graph for *m^yag ("magician"): both are written ☩."



T45:2 T45:6

Figure 74. Two Carved Heads (T45:2 and T45:6) from Chou-yüan, Shensi Province, China. From Yin Sheng-p'ing, *Wen-wu*, 1986. 1, 46–49

163 Needham and Robinson, *Science and Civilisation*, 135.

164 Yin Sheng-p'ing, "Investigation of the Racial Affinity of Two Western Chou Human Heads Sculpted of Shell," *Wen-wu* (Cultural Relics) (1986): 46–49.

165 V. Mair, "Old Sinitic " M^yag," Old Persian "Maguš," and English "Magician," *Early China* 15 (1990): 40.

Mair suggests that the English word "magician" or "mage" and Modern Standard Mandarin *wu* (巫) ultimately derive from the same Indo-Iranian word: *magus*, which comes to English via Latin: *magus* (pl. *magi*).[166] The theory is that the Magi,[167] the priests of Zoroastrianism, who traveled widely, were, in fact, active in ancient China. As evidence of this, Jao Tsung-i cites the fact that "Chinese texts dating from or describing the situation during the Eastern Chou [770–256 BC] show that the m^yag were present in nearly all of the contending states."[168]

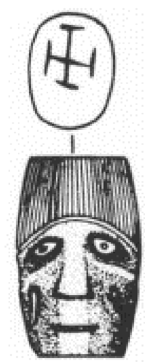


Figure 75. Drawing of Head T45:6, showing carved symbol on top of head

The earliest appearance of the character *wu* (巫) is on oracle bones that date to the 1250 BC. G. Boileau[169] suggests four possible meanings of this character (巫), as used on the bones:

- a spirit — *wu* — of the north or east, to which sacrifices are offered
- a sacrifice, possibly linked to controlling the wind or meteorology

166 The earliest languages known to have been used in Xinjiang — Saka and Tocharian — are both from the Indo-European language family (J. P. Mallory, "Bronze Age Languages of the Tarim Basin," *Expedition* 52 [3] [2010]: 44–53). Furthermore, "there is massive linguistic evidence of Sinitic and Indo-European cultural exchange from at least the late Neolithic...." (V. Mair, "Old Sinitic 'M^yag,' Old Persian 'Maguš,' and English 'Magician,'" *Early China* 15 [1990]: 43.)

167 Incidentally, a Vatican text written in the eighth century AD, in Syriac — a language that emerged during the first century AD in Upper Mesopotamia — makes reference to twelve Magi. Even today, in Eastern Christianity, especially in the Syriac churches, the Magi often number twelve. http://www.aina.org/ata/20101203200558.htm

168 Mair, "Old Sinitic 'Myag'" (referencing Jao Tsung-I, "New Light on *wu*"): 39.

169 Boileau, "Wu and Shaman," *Bulletin of the School of Oriental and African Studies* 65 (2) (2002): 350–378.

- an equivalent for *shi* 筮, a form of divination using achillea (a plant)
- a living human being, possibly the name of a person, tribe, place, or territory

By the Chou and Han eras, the *wu* (shaman/magi) was most often associated with rain making rituals — and "[t]here is ample evidence that the potency of rainmaking rituals in the Chou and Han periods was understood in terms of resonance between things of like kind."[170]

> Rainmaking often involved ritual exposure, either of a woman[171] as is seen in Shang and Chou sources, a shaman (*wu* 巫) as was more common in the Han, or a Buddhist or Taoist monk or even the emperor, as is recorded throughout medieval times. In such ritual exposure the supplicant (or victim) is exposed to the sun, sometimes naked, thereby subjecting his or her body to the same adverse effects suffered by the parched earth in times of drought.[172]

Sharf compares the views of Needham and Schafer regarding the belief system that underlies the rain making ritual:

> Schafer tends toward the anthropomorphic: the suffering of the exposed priest or sacrificial victim impresses upon heaven the pain of heat and drought. Heaven is moved (or coerced) to respond compassionately and cause rain to fall. Needham takes a more naturalistic view and understands rainmaking as an exercise in the manipulation of cosmic forces along the lines of five-phase theory and alchemy.[173]

170 Sharf, "Chinese Buddhism," 87.

171 Evidence suggests that in earliest times the *wu* (巫) were predominantly female.

172 Sharf, "Chinese Buddhism," 86.

173 Sharf, "Chinese Buddhism," 87.

To summarize, from the time of the Shang dynasty the concept of sympathetic resonance was evident in China, in an early form. Yet there is also evidence, at this time, of the *wu* (magi), who may have brought these ideas from the West. It is important to note, therefore, that a 2005 paper, published by the Shanghai Conservatory of Music, describes evidence of Near Eastern shamanic (i.e., Magian) presence in the Xinjiang graves that contain angular harps, mentioned earlier:

> in Shanshan graves, bows of Sassanid character, clothes of Shaman traits and skulls of Caucasus race have also been found. All those cultural relics have shown clearly that ... cultural communication had already begun between Xinjiang and the areas of Altai, Assyria and the Black Sea.... [T]he prevalent view of Chinese *konghou*'s origin [i.e., that the *konghou* (harp) was invented in China] is questionable.[174]

The Sassanian Empire, centered in Persia, existed from 224 to 651 AD. Therefore, the linking, in the above quotation, of harps dating to 3000 BC with the Sassanids is anachronistic. Nevertheless, what the author implies is an association with Zoroastrianism, the Sassanid religion.

The Chinese refer to Zoroastrianism as *Bàihuǒjiào* (拜火教), literally "the doctrine of fire worship," for, as described by M. Boyce, "the basic objects of the Zoroastrian cult are ... those of the ancient Stone Age pastoralists, namely water and fire."[175] These two elements became linked with the gods Varuna and Mithra, respectively. Furthermore, Mithra was associated not only with fire, but with the sun: "Mithra ... lord of fire ... was believed to accompany the sun, the greatest of all fires, in its daily course across the sky...."[176] Remember too that the Roman Mithraic ladder shows reverence for the sun by placing it — and consequently the number 4 — at the highest level.

174 https://web.archive.org/web/20051120100750/http://musicology.cn/Article/ytdt/conference/200506/299.html

175 Boyce, *Zoroastrians*, 3.

176 Boyce, *Zoroastrians*, 9.

Figure 76. Angular Harp with Camel Carving from Grave M63:1, Hami Wupu Eschernan Cemetery, Xinjiang. Photo: He Zhiling and Wang Yongqiang, "*Konghou* in Hami," *Chinese Music* (2018): 119

Further evidence of a link between the Xinjiang harps and Zoroastrianism is given by He Zhiling in a recent doctoral thesis[177] detailing a total of twenty-three harps, excavated prior to 2018. One harp, excavated in 2010 from Hami Wupu Eschernan Cemetery, Xinjiang,[178] dating to the sixth or seventh century BC, is particularly interesting. This harp has a carving of a two-humped camel at the end of its quite damaged resonance box (Figure 76). The two-humped or Bactrian camel was domesticated circa 2500 BC, east of the Zagros Mountains (which mark Iran's western border), with the practice then moving into Mesopotamia.

In summary, according to both Western and Chinese scholarship, the Xinjiang angular harps are linked with the religious traditions of the Ancient Near East. As we saw earlier, the religious traditions that arose in the Near East, as early as the second millennium BC, had the fundamental ingredients of correlative cosmology. Remember, for example, the chief lamentation singer who "for seven days and seven nights put in place seven *balangs*, like the firm base of heaven." Here, we see the linking of number (seven); music (*balang* = arched harp); and cosmic order ("firm base of heaven"). Another example is that on tablet CBS 1766, the name of one of the strings is "fourth, small / string created by Ea" (line 4,

177 He Zhiling and Wang Yongqiang 贺志凌 王永强, "The Musical Archaeological Study of *Konghou* in Hami Wupu Eschernan Cemetery," *Chinese Music* (2018): 117–122. (English translation forthcoming in *Sino-Platonic Papers*.)

178 Wang Yongqiang and Dang Zhihao 王永强 党志豪, "A New Archaeological Discovery at Hami Wupu Eschernan Cemetery, Xinjiang," *The Western Regions Studies* (2011): 134–137.

Table 13). Ea is the Babylonian god of knowledge, and one of "the seven gods who decree."[179] In Babylonian texts, Ea is represented by a goat with a fish's tail, which is, in turn, linked with the constellation Capricorn (the constellation which, according to Porphyry, is the celestial gate through which souls ascend). Here, we see the linking of music ("fourth string"); god (Ea); and cosmos (Capricorn). If we remember, too, that from as early as 600 BC, the planets were listed by the Babylonians sequentially, in non-astronomical order, we have all the components of correlative cosmology.

In light of the material presented in this section, the following questions arise:

Is the Chinese tradition of correlative, musico-cosmological thought an independent invention, or was it influenced by a similar tradition, already established in the West? And if this tradition was transmitted from the west, what was the date of transmission? At the beginning of the first millennium BC, with harps that entered Xinjiang? Or at a later date?

Let us now review the material presented in this paper and attempt to draw some conclusions regarding the possible transmission, from the Near East, of both the musical and musico-cosmological systems of ancient China.

179 S. N. Kramer, *The Sumerians: Their History, Culture, and Character* (University of Chicago Press, 1963), 123.

CONCLUSION

CONCLUSION

In the Introduction, reference was made to the hypothesis, put forth in 1962 by Joseph Needham and Kenneth Robinson, that the musical systems of ancient China and ancient Greece have a common origin: Mesopotamia. In this paper we have looked only briefly at the connection between the musical systems of the West and Mesopotamia, for much of the research in this area was done — between 1960 (the date of the discovery of the first cuneiform tablets relating to the Mesopotamian tonal system) and the present — by other scholars in the field.[180] (The contributions made by the author of this paper are, specifically, (1) suggesting that the heptagram drawn on CBS 1766 is not regular, but that its points indicate seven of twelve equidistant points on a circumscribed circle, (2) showing that such a heptagram exactly illustrates the re-tuning cycle described on Mesopotamian tablet UET VII 74, and (3) showing that this irregular heptagram and the sequence 4,1,5,2,6,3,7 are both derived from simple harmonic ratios, using the ancient Chinese *sanfen sunyi* method.) It has been the focus of this paper, therefore, to outline the similarities between the musical systems of Mesopotamia and ancient China, and also to suggest that the tradition of "correlative cosmology," often associated exclusively with ancient Chinese philosophical traditions, may have had, in fact, a Near Eastern origin, and may have disseminated both to the West and China. Let us now draw some conclusions regarding what has been discussed.

Archaeological evidence proves that Caucasoid peoples having either a direct or indirect link to the Near East established permanent settlements in Xinjiang as early as 1800 BC. By circa 1000 BC, small angular harps of Mesopotamian design had arrived in Xinjiang. The musical system associated with these harps in Mesopotamia — a system of seven diatonic tunings, in use for over a thousand years, from at least 1800 BC — was a simple mnemonic system that could have easily been transmitted with the harps, without the accompaniment of textual instruction.

Although there is no textual proof of the transmission of the Mesopotamian tonal system to Xinjiang and, from there, to the East Asian Heartland, cuneiform tablets and ancient Chinese texts nevertheless indicate strong similarities between the two systems. For, as we have seen, the *Lǚshì*

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180 A number of academic papers that discuss the Mesopotamian tonal system are available here: https://musicircle.net/academic-papers-on-the-mesopotamian-tonal-system/

chūnqiū (239 BC) describes the use of the *sanfen sunyi* method to generate twelve pitches — the *shí'èr lǜ* — and these pitches are identical to those generated by the re-tuning cycle described on UET VII 74 (1800 BC). Moreover, as described in the *Guanzi* (fifth–first century BC), the *sanfen sunyi* method was used to create scales having fewer than twelve notes, most notably a pentatonic scale. Yet, as evidenced by sets of *bianzhōng* bells dating to the first half of the first millennium BC, and later textual references, heptatonic scales generated by the *sanfen sunyi* method were also in use — scales that are identical to two of the Mesopotamian tunings.

As we have seen, only two diatonic scales can be mathematically generated using the *sanfen sunyi* method — a scale with an up-generated fundamental, and a scale with a fundamental that is not up-generated. These are the two scales used by the ancient Chinese that resemble two of the Mesopotamian tunings: *išartum* and *qablitum*. (The other five Mesopotamian tunings are inversions of these two.) Moreover, tablet UET VII 74 suggests that the Mesopotamians had knowledge of the Pythagorean comma — knowledge that is also indicated by the instructions in the *Lǚshì chūnqiū* that describe the *sanfen sunyi* method.

To date, there is no proof that the Mesopotamians used a pentatonic scale. Nevertheless, scales that are constructed from consecutive fifths — as scales generated by the up-and-down principle are — are subsets of each other. Consequently, the Mesopotamian re-tuning cycle could have been used, quite easily, to re-tune a cycle of pentatonic modes, or even the modes of scales having fewer than five notes. Archaeological evidence seems to suggest this possibility for, as we have seen, several Mesopotamian artifacts depict instruments having fewer than seven strings.

We have seen that the there is no explicit mention, in cuneiform texts, of the Mesopotamian tonal system having a mathematical basis. Nevertheless, the re-tuning instructions on tablet UET VII 74 can be illustrated *exactly* by a seven-pointed star that is derived, mathematically, using the ancient Chinese *sanfen sunyi* method. Moreover, the derived seven-pointed star resembles the heptagram that is drawn on tablet CBS 1766 (1780–600 BC).

Written on tablet CBS 1766, UET VII 74, and CBS 10996 (700 BC), are inversions of the sequence 4,1,5,2,6,3,7 — a sequence that is generated when the up-and-down principle is used to create the *shí'èr lǜ* pitches. As we have seen, inversions of this sequence also select the notes in the seven modes of the diatonic scale from the circle of fifths — and it is this application of the sequence that forms the basis of the re-tuning cycle described on UET VII 74.

As was documented in the second section of this paper, the sequence 4,1,5,2,6,3,7 was also used, in the West, to create a complex system of musical analogies: the spacings of the spheres, the value system of the metals, the Mithraic soul-ladder, and the seven-day week. Evidence suggests that this theological/philosophical system had a Near Eastern origin for not only is this sequence fundamental to the Mesopotamian tonal system, when discussing the week-day order, Celsus examines "the reason of the stars being arranged in this order … [and gives] … musical reasons … quoted by the Persian theology."

For over a millennium, from 600 BC to 650 AD, the state religion of Persia (ancient Iran) was Zoroastrianism. Yet many scholars believe that the roots of Zoroastrianism may reach back to the second millennium BC. This early date is supported by the fact that the Xinjiang harps, dating to the early first millennium BC, are found in graves that contain artifacts that show a connection to Zoroastrianism.

Moreover, according to recent evidence,[181] it may be possible to trace the Magi, the priests of Zoroastrianism, as far back as 1100 BC, living near what is, today, the border between Iran and Turkmenistan. As V. Mair notes, this dating for the Magi — circa 1100 BC — is "a period of time compatible with the *m^{y}ag* who are mentioned fairly frequently in the oracle bone inscriptions."[182] We must consider, therefore, that the Magi may have reached China as early as the late second millennium BC.

As previously mentioned, it is believed that the Mesopotamian tonal system was used in Elam (later Persia) from at least 1800 BC. Furthermore, according to M. Boyce the concept of the postmortem ascent of the soul probably originated in ancient Iran, sometime before 1800 BC, when the proto-Indo-Iranians drifted apart to become the Indians and the Iranians.

It is proposed here that by the early first millennium BC, the Zoroastrian tradition began to describe the ascent of the soul using "musical reasons" drawn from the Mesopotamian tonal system and

181 V. Sarianidi, "Togolok 21, an Indo-Iranian Temple in the Karakum," *Bulletin of the Asia Institute* 4 (1990): 159–165.

182 Mair, "Old Sinitic 'M^{y}ag,' Old Persian 'Maguš,' and English 'Magician'," 37–38.

that this concept evolved into a fundamental magico-religious belief. This belief was then disseminated to other cultures and was applied, either by the Zoroastrians themselves or by the cultures that they influenced (via the traveling Magi) to generate/describe the value system of the metals, the spacings of the spheres, the Mithraic soul-ladder, and the seven-day week.

As we have seen, this Western system of musical analogies is an example of correlative cosmology whereby associations are made between items having the same relative position within their respective sequences (for example: the sun; Sunday; gold). Yet, until recently, this term — correlative cosmology — was used exclusively to describe an ancient Chinese tradition, for, as we have seen, the ancient Chinese, too, devised a complex system of correspondences. As with the Western tradition, the Chinese made use of musical analogy: the concept of *ganying* was used to explain the affinity between things of the same category. This concept is first mentioned in Chinese texts circa 400 BC, evolving into a complex cosmological system around the second century BC.[183]

It is proposed, here, that the phenomenon of correlative cosmology, which draws affinities between items in different categories by placing them in the same sequential positions — while at the same time being a system that relies on musical analogies — is too extremely unusual an invention to have developed, independently, in two different places. Rather, it is suggested here that this phenomenon originated in the Near East and was transmitted, in a still developing form, to China, during the second half of the first millennium BC. This is not to say that the idea of "like affects like" — as seen in the *wu* rainmaking rituals — could not have existed, independently, in pre-historic China. Rather, *it is to suggest that the co-usage of the concepts of musical analogy and sequential placement to describe cosmic order is a uniquely Near Eastern invention that was later adopted in China*.

Let's take a moment to imagine the very first thought processes that led to the evolution of the concept of correlative cosmology. We have seen that, as early as 1800 BC, the Mesopotamians were aware of the twelve pitches generated by the cycle of fifths. On the same tablet that documents this knowledge

183 As we saw, the systems of correlative cosmology in both China and the West took time to become standardized. For example, in China, many variations of the five-phase sequence are given in pre-Han and Han texts until, eventually, two specific orders became the norm. The situation was similar in the West. For example, Celsus linked copper with Jupiter, while according to most later sources it was associated with Venus. Similarly, Plato gave the positions of the sun, Venus and Mercury slightly differently than they were given in the later Ptolemaic model.

(UET VII 74), seven heptatonic tunings are described: the seven-note diatonic modes. Here, we already see the numbers 7 and 12 playing an important role in the Mesopotamian tonal system. As we have seen, these numbers are not drawn, randomly, from the imagination: they have a mathematical basis — the seven-number sequence 4,1,5,2,6,3,7 and the twelve-pitch circle of fifths are generated from the interplay of fractions 1/2 and 2/3 — two fractions that correspond respectively to the intervals of the octave and the fifth.

The numbers 7 and 12 also played a role in Mesopotamian astronomy: seven classical planets were observed, while the stars that lie along the ecliptic were divided into twelve constellations (which the Greeks later called the "zodiac"). The counting of seven "planets" can be seen as a logical, rational act: the Mesopotamians saw seven "planets." But the division of the ecliptic into twelve is arbitrary. Some scholars attribute this division to a recognition, by the Mesopotamians, that there are 12.37 cycles of the moon in a year. (Moreover, as discussed earlier, the number 12 also has associations with the orbital period of Jupiter.) But there is another possible reason for the division of the ecliptic into twelve — a *musical* reason.

The twelve-part division of the ecliptic by the Babylonians came relatively late in Mesopotamian history — at the end of the fifth century BC. Yet, as we have seen, the number 12 is already apparent in the Mesopotamian tonal system by 1800 BC. Considering what we have seen regarding the cosmological application of the sequence 4,1,5,2,6,3,7, it is not unreasonable to imagine that the division of the ecliptic into twelve was based on a musical analogy. Or rather, that the numbers 7 and 12 — seemingly inherent in both music and astronomy — inspired the two subjects to become fused into one system, at once secular and sacred; practical and cosmological — the remnants of which came down to us, in the West, as the music of the spheres.

Earlier, we discussed the phenomenon of the octave and the fact that each higher octave is heard by halving a string's length. Furthermore, because something can be divided in half, indefinitely, a single string contains an infinite number of octaves.[184] We also saw that an infinite number of fifths can be generated. Our musical system, inherited from the Mesopotamians, is essentially derived from the interplay of these two cycles.

184 This is, essentially, the dichotomy paradox of the Greek philosopher Zeno (490–430 BC).

Yet music is not only cyclical, but linear: the string itself resembles a line — and within this line, from one end to the other, is embedded the infinitely repeating sequence 4,1,5,2,6,3,7. This sequence, as we have seen, was used by the Mesopotamians to create their re-tuning cycle. Then, almost 2000 years later, it was documented by Celsus as being used in the Mithraic mysteries, to represent hierarchical levels: a vertical ladder (*scala*). Sometime during this 2000-year span we can imagine the birth of this new concept: that of graduated levels — a phenomenon that is, in fact, what we perceive, subconsciously, when we listen to a scale. It is proposed, here, that this concept profoundly changed the way the ancient Persians thought about their world. The universe (the classical planets; the metals; even time itself) became imagined as having a graduated or sequential order, but this order was not visible: instead, it was based on the invisible patterns of music.

Was it the proto-Indo-Iranian idea of the Soul Journey that inspired this concept of cosmic hierarchy? Or was the concept of hierarchy simply applied, later, to describe celestial ascent of the soul? We cannot know. But what is certain is that the idea that the soul can ascend to the heavens by climbing through seven levels became widely disseminated across Eurasia and beyond.

As Eliade describes, the concept of the postmortem ascent of the soul is tied to the shamanic practice of ritual ascent, whereby the shaman visits the celestial realm, while still living. For one result of the shaman's journey is that s/he returns to earth with information that helps map the "funerary geography" through which the soul, after death, must pass:

> … the shaman has been able to contribute decisively to the *knowledge of death*. In all probability many features of "funerary geography", as well as some themes of the mythology of death, are the result of the ecstatic experiences of shamans.… The unknown and terrifying world of death assumes form, is organized in accordance with particular patterns; finally it displays a structure and, in the course of time, becomes familiar and acceptable.[185]

185 Eliade, *Shamanism*, 509–510.

As we have seen — with the Mithraic ladder ritual and with Plato's Myth of Er — the analogy of music was often used in conjunction with the concept of the soul's ascent. This is also the case in many different shamanic traditions, where music enables/accompanies the celestial journey:

> … he [the Ugrian shaman (Siberia)] is journeying in ecstasy through distant regions, but he is not unconscious … the basic experience is ecstatic, and the principal means of obtaining it is, as in other regions, magico-religious music.[186]

What form did this magico-religious music take? What were the instruments that it was played on? Evidence shows that the first instruments used by Eurasian shamans were drums. Later, when stringed instruments were introduced, they had a significant impact. This is shown, for example, by the name that the Yurak shamans of Siberia gave to their drums, for "the idea of ecstatic journey is … found in the name that the shamans of the tundra Yurak give their drum: *bow* or *singing bow*."[187] Furthermore, in some traditions the drum was replaced by a stringed instrument, whose role, like the drum, was to aid the shaman in the ecstatic journey:

> The Kirgiz *baqça* [shaman of Central Asia] does not use the drum to prepare the trance, but the *kobuz*, which is a stringed instrument. And the trance, as among the Siberian shamans, is induced by dancing to the magical melody of the *kobuz*. The dance … reproduces the shaman's ecstatic journey to the sky … the magical music … is one of the many ways of undertaking the ecstatic journey or ensuring its success.[188]

What we see here supports the earlier hypothesis: that at a certain moment in history, the soul's ascent became described — or experienced — as having a musical dimension. And this musical

186 Eliade, *Shamanism*, 223.

187 Eliade, *Shamanism*, 174.

188 Eliade, *Shamanism*, 175.

dimension, although attainable through different sorts of musical expression — the Siberian shaman's drum, for example — came, eventually, to be epitomized by the musical string.

As we have seen, the sequence 4,1,5,2,6,3,7 repeats indefinitely — generated by the infinite number of octaves and fifths contained within a single string. No wonder, then, that the numbers of music took on such cosmic importance. For the division of the unmappable territory of the octave into smaller, manageable steps, must have suggested that the heavens — the "funerary geography" — too, could be navigated: that a ladder (*scala*) could be set up to heaven and that the steps of this ladder — the sequence 4,1,5,2,6,3,7 — would enable the soul to achieve its ascent.

Were the Siberian shamans, "dancing to the magical melody of the *kobuz*", aware of this magical/musical mathematics? We cannot know. But we do know that the ancient Chinese had knowledge of the *sanfen sunyi* method — which generates the sequence 4,1,5,2,6,3,7 — and that they used musical analogies to describe the workings of the cosmos. One passage from the *Guo Yu* (fifth century BC) that recounts a discussion between a king and his counsellor stands out:

"The king said, 'What is meant by the seven pitches?' [王曰:'七律者何？']"[189] The counsellor, instead of giving a musical answer, describes the positions of the sun, the moon, Jupiter, and Mercury during the battle of Muye — the battle that established Wu as the first king of the Zhou dynasty. The counsellor recounts that Wu studied the positions of these bodies, prior to the battle, and also took into consideration the celestial position of an up-coming sun-moon conjunction. The counsellor describes these positions relative the twelve *ci* (each *ci* being one twelfth of the ecliptic), and also relative to the 28 *xiù* (宿) (a different division of the ecliptic into 28 "Lunar Mansions"). After describing these five planetary positions (those of the sun, the moon, Jupiter, Mercury, and the up-coming sun-moon conjunction), the counsellor explains that:

> King Wu wished to combine these five positions and three locations and utilize them [to be victorious in battle]. From *Shun huo* to *T'ien Ssu* [two specific areas of the ecliptic] there are seven ranks. The divisions from south to north are the identical seven. In all things men and the spirits make them accord by means of number and proclaim them

189 *Chou Yü* III, Section 7, trans. J. Pinckney Hart, Jr., in "The Philosophy of the Chou Yü" (1973), 398.

> with sounds (music). Only after the numbers are made to accord and the musical notes are harmonized will things be unified. Therefore, the number seven is used to unify their number, and the standard pitches are used to harmonize the notes. And thus it is that there are seven pitches. [190]
>
> 王欲合是五位三所而用之.自鶉及駟七列也.南北之揆七同也，凡人神以數合之，以聲昭之.數合聲和，然後可同也.故以七同其數，而以律和其聲，於是乎有七律.

We can understand from this text that the number seven had both musical and astronomical significance to the ancient Chinese. We have seen the importance of the number seven with regards to music. But what is its importance in astronomy? As mentioned before, there were seven known "planets" in the ancient world. But the "positions" and "locations" mentioned in the text suggest that something else is being discussed. What do these two terms refer to and what is their relationship — alluded to in the text — to the number seven?

To answer this, it must be explained that when the ecliptic is divided into twelve constellations, or *ci*, a seven-step pattern is generated. This pattern is best illustrated by the changing elevation of the sun, over the course of a year. On winter solstice, the sun is low in the sky. As the year progresses, its declination increases until, at summer solstice, it is at its highest elevation. During the next half of the year the sun's elevation decreases, until it reaches, once again, its lowest elevation at winter solstice. In other words, one zodiac constellation (or *ci*) "contains" the sun on summer solstice; the *ci* directly opposite it (i.e., six positions around the zodiac circle) "contains" the sun on winter solstice. The remaining ten *ci* can then be imagined as five pairs — because pairs of two of the remaining ten *ci* have the same elevation.

There is evidence that this tradition of conceptualizing the zodiac of twelve constellations as being related to a seven-step "ladder" existed in the West. Panagiotidou and Beck, in discussing Porphyry's description of the Zoroastrian cave that bore the image of the cosmos (see footnote 101), note

190 *Chou Yü* III, Section 7, trans. J. Pinckney Hart, Jr., in "The Philosophy of the Chou Yü" (1973), 398.

that the "climates" referred to by Porphyry are related to the ancient tradition of dividing the distance between the north and south poles into climactic zones — usually seven in number — called *clima*. In discussing the relationship between the twelve zodiac constellations and the *clima*, Panagiotidou and Beck note that between the extremes of summer solstice and winter solstice "the zodiac signs formed "isodynamounta" (equally powered) … pairs" which determined the intermediate northern and southern *climata* [pl. of *clima*].[191]

In other words, one zodiac constellation (or *ci*) marks the top of the sun's path and one *ci* marks the bottom. The five positions in between — each associated with a pair of *ci* — mark the ascending/descending journey (Figure 77). In this way, a circle of twelve — the twelve *ci* or zodiac constellations — becomes a ladder of seven: the seven *clima*.

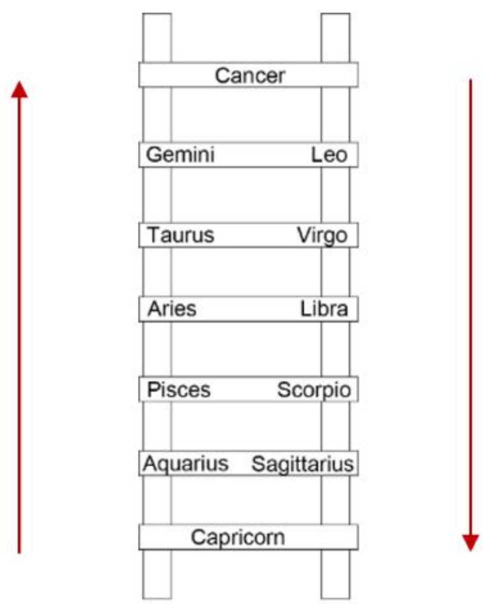

Figure 77. Ladder of Seven Clima (astronomically correct from circa 2000 BC to circa 0 BC)

191 O. Panagiotidou and R. Beck, *The Roman Mithras Cult: A Cognitive Approach*. Scientific Studies of Religion: Inquiry and Explanation (London; New York: Bloomsbury Academic, 2017), 101.

The Latin term *clima* is derived from the Proto-Indo-European root **klei*, meaning "lean" which entered Latin via the Greek word *klima* — which is rooted in the verb κλίνειν (*klinein*) meaning "to slope, lean, or incline." Other Greek words derived from the root **klei* are *kline* ("bed"), *kleitoris* ("clitoris"), and *klimax* ("ladder").

Tradition suggests that the Greek *klimax* ("ladder") was imagined as having seven levels. For the related term *klimaktērikós*, a word also derived from the root **klei*, refers to critical years in a person's life that were thought to occur every seven years. A related tradition is the sabbatical — a year of rest given to an academic once every seven years. The word sabbatical is cognate with the Greek word *sabbatikos* ("rest") and the Hebrew word *shabbat* ("cease" or "stop"), from which the term Sabbath is derived. Both these terms come from the Sumerian term *sa-bat* ("mid-rest").

As we have seen, a parallel pattern — of a circle of twelve generating a ladder of seven — is found in music: the circle of twelve fifths generates the seven-number sequence 4,1,5,2,6,3,7, which, in turn, generates the diatonic scale (*scala*). Is it this shared pattern that accounts for the fact that when King Wu's councillor is asked to explain what is meant by the seven pitches,[192] he gives an astronomical answer? This is plausible, for remember that the Mithraic ladder — on which the sequence 4,1,5,2,6,3,7 is encoded — is at once both astronomical and musical.

As mentioned earlier, Eliade believes that the widespread belief in seven cosmic levels is due to Mesopotamian influences and was inspired by their knowledge of the seven classical planets. Yet, as we have seen, from at least 1800 BC the Mesopotamians associated the number 7 with both the cosmos and music. Is it possible, therefore, that the Mesopotamian fascination with the number 7 was inspired not only by the study of astronomy, but by the study of music?

It was previously suggested that it was during centuries of cultural exchange between Persia and Mesopotamia that the patterns embedded in the Mesopotamian tonal system became incorporated into what Celsus refers to as the "Persian theology" and that this tradition was disseminated by the Magi. Here, then, is the possible means of transmission of the musical and musico-cosmological systems of the Near East to China. The proposed time line is as follows:

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192 Recall that Wei Zhao, commenting on this passage about seven hundred years after it was written (see footnote 59), identifies the "seven pitches" as a diatonic scale that is identical to the Mesopotamian tuning *qablītum*.

First, the Mesopotamian tonal system — in use in Mesopotamia from at least 1800 BC — arrived in Xinjiang with angular harps of Mesopotamian design circa 1000 BC. As Lawergren suggests, the angular harps then metamorphosized into the *qin*, spreading down the Hexi Corridor, during the first half of the first millennium BC, into the East Asian Heartland. As previously suggested, the mathematical underpinning of this system (i.e., the up-and-down principle) was not transmitted immediately with the knowledge of how to play and tune these harps/*qin*, for proof of knowledge of this principle is not found until circa 433 BC, with the bells of Marquis Yi.

It is proposed that concurrent with the transmission of the knowledge of the mathematics of music in the late sixth century BC or the early fifth century BC, the Near Eastern system of musico-cosmological/correlative thought — which, it is suggested here, coalesced in Persia in the early first millennium BC — was transmitted to the heartland of China. For it is during this period that the idea of *ganying* or "correlative resonance" first appears in ancient Chinese texts to denote a cosmological principle that is described, analogically, using the principles of music.

In conclusion, we have seen that there is archaeological proof that the Mesopotamian angular harp arrived in the Tarim Basin circa 1000 BC. Yet no definite proof exists concerning the transfer of the Mesopotamian tonal system to China or that the musico-cosmological system of ancient China was influenced by a similar tradition already present in the Near East. For if these two systems were, in fact, transmitted from the Near East, they were carried invisibly with the angular harps that made their way across the Steppes of Central Asia to the Tarim Basin, and from there, via the Hexi Corridor, into the heartland of China.

BIBLIOGRAPHY

Aaboe, Asger. *Episodes from the Early History of Astronomy*. New York: Springer Verlag, 2001.

Aristotle. *Metaphysics*. Translated by W. D. Ross. Buffalo: Prometheus Books, 1991.

Bagley, Robert. "Ancient Chinese Bells and the Origin of the Chromatic Scale." *Zhejiang University Journal of Art and Archaeology* 2 (2015): 56–102.

Bagley, Robert. "The Prehistory of Chinese Music Theory." *Proceedings of the British Academy* 131 (2005): 41–90.

Bloomfield, M. W. "The Origin of the Concept of the Seven Cardinal Sins." *The Harvard Theological Review* 34 (2). Cambridge: Cambridge University Press (1941): 121–128.

Bodde, Derk. "The Chinese Cosmic Magic Known as Watching for the Ethers." In *Essays on Chinese Civilization*, 351–352. Princeton: Princeton University Press, 1982.

Boileau, G. "Wu and Shaman." *Bulletin of the School of Oriental and African Studies* 65 (2) (2002): 350–378.

Bousset, W. *Himmelsreise der Seele* (*Sonderausgabe*). Darmstadt: Wissenschaftliche Buchgesellschaft, 1960.

Boyce, Mary. *Zoroastrians: Their Religious Beliefs and Practices*.London: Routledge/Kegan Paul, 2001.

Callahan, W. A. "Discourse and Perspective in Daoism: A Linguistic Interpretation of Ziran." *Philosophy East and West* 39 (2) (1989): 171–189.

Cassius Dio. *Roman History*. Translated by Ernest Cary. London: Heinemann, 1914.

Chen, C.-Y. *Early Chinese Work in Natural Science: A Re-examination of the Physics of Motion, Acoustics, Astronomy and Scientific Thoughts*. Hong Kong: Hong Kong University Press, 1996.

Cook, Scott. "Music in the Zhou." In *Oxford Handbook of Early China*, 479–488. Oxford: Oxford University Press, 2020.

Cumont, Franz. *The Mysteries of Mithra*. Translated by T. J. McCormack. Chicago: Open Court, 1903.

Drower, Ethel S. *The Mandaeans of Iraq and Iran: Their Cults, Customs, Magic Legends, and Folklore*. Oxford: Oxford University, The Clarendon Press, 1973.

Eberhard, Wolfgram. "Beiträge zur Kosmologischen Spekulation Chinas in der Han-Zeit." *Baessler-Archiv* 16 (1) (1933): 1–100. (Berlin, Universität, Dissertation, 1933).

Eliade, Mircea. *Shamanism: Archaic Techniques of Ecstasy*. Princeton: Princeton University Press, 2004.

Franklin, John C. "Lyre Gods of the Bronze Age Musical Koine." *Journal of Ancient Near Eastern Religions* 6.2 (2006): 39–70.

Franklin, John Curtis. 2016. *Kinyras: The Divine Lyre.* Hellenic Studies Series 70. Washington, DC: Center for Hellenic Studies.

Friberg, Joran. "Seven-Sided Star Figures and Tuning Algorithms in Mesopotamian, Greek, and Islamic Texts." *Archiv für Orientforschung* 52 (2013): 121–155.

George, A. R. *House Most High: The Temples of Ancient Mesopotamia*. Winona Lake, IN: Eisenbrauns, 1993.

Gnoli, G. "Politica religiosa e concezione della regalità sotto gli Achemenidi." In *Gururājamañjarikā. Studi in onore di Giuseppe Tucci, Napoli*, 1974.

Gregory of Nyssa. *On the Sixth Psalm, Concerning the Octave*. Leiden: E.J. Brill, 1962.

Guanghui Dong, Yishi Yang, Xinyi Liu, Haiming Li, Yifu Cui, Hui Wang, Guoke Chen, John Dodson, and Fahu Chen. "Prehistoric Trans-continental Cultural Exchange in the Hexi Corridor, Northwest China." *The Holocene* 28 (issue 4, 2018): 621–628. First published online October 17, 2017.

Gurney, Oliver. R. "An Old Babylonian Treatise on the Tuning of the Harp." *Iraq* 30 (1968): 229–233.

Gurney, Oliver. R. "Babylonian Music Again." *Iraq* 56 (1994): 101–106.

Han shu, *The History of the Former Han Dynasty*. Translated by Homer H. Dubs. Baltimore: Waverly, 1938–1955.

He Zhiling and Wang Yongqiang. "The Musical Archaeological Study of *Konghou* in Hami Wupu Eschernan Cemetery." *Chinese Music* (2018): 117–122.

Herodotus. *The History*. Translated by David Grene. Chicago: University of Chicago Press, 1987.

Horowitz, Wayne. *Mesopotamian Cosmic Geography*. Winona Lake, IN: Eisenbrauns, 1998.

Iamblichus. *The Life of Pythagoras*. Translated by T. Taylor. Illinois: Theosophical Publishing House, 1918.

Jeremias, Alfred. *Handbuch der alterorientalischen Geisteskultur — Bücher gebraucht*, 197–199. Berlin and Leipzig: Verlag von Walter de Gruyter, 1929.

Jung, Carl G. Forward to the *I Ching*. Translated by R. Wilhelm. Princeton: Princeton University Press, 1950.

Knoblock, John, and Riegel, Jeffrey. *The Annals of Lü Buwei: A Complete Translation and Study*. Stanford: Stanford University Press, 2000.

Kramer, S. N. *The Sumerians: Their History, Culture, and Character*. University of Chicago Press, 1963.

Krispijn, Theo J. H. "Beiträge zur altorienalischen Musikforschung I." *Akkadica* 70 (1990): 1–27.

Lawergren, Bo. "The Origin of Musical Instruments and Sounds." *Anthropos* Bd. 83, H. 1./3. (1988): 31–45.

Lawergren, Bo. "Strings." In *Music in the Age of Confucius*, edited by Jenny F. So, 65–85. Washington, DC: Freer Gallery of Art, 2000.

Lawergren, Bo. "Western Influences on the Early Chinese Qin-Zither." *Bulletin of the Museum of Far Eastern Antiquities* 75 (2003): 79–109.

Lawergren, Bo. "Angular Harps Through the Ages, a Causal History." In *Studien zur Musikarchäologie VI*, edited by Arnd Adje Both, Ricardo Eichmann, Ellen Hickmann, and Lars-Christian Koch. *Orient-Archäologie 22* (Rahden, 2008): 261–281.

Le Blanc, C. *Huai-nan Tzu: Philosophical Synthesis in Early Han Thought: The Idea of Resonance* (*Kan-Ying*). Hong Kong: Hong Kong University Press, 1985.

Li Chunyi. *Zhongguo shanggu chutu yueqi zonglun*. Beijing: Wenwu chubanshe, 1996.

Li Chunyi. *A History of Pre-Imperial Chinese Music*. Beijing: Renmin yinyue chubanshe, 2005.

Lloyd, Geoffrey Ernest Richard. *Adversaries and Authorities: Investigations into Ancient Greek and Chinese Science*. Cambridge: Cambridge University Press, 1996.

MacKenzie, D. N. "Zoroastrian Astrology in the *Bundahišn*." *Bulletin of the School of Oriental and African Studies* 27 (1964): 511–529.

Mair, Victor H. "Old Sinitic 'M^{y}ag,' Old Persian 'Maguš,' and English 'Magician.'" *Early China* 15 (1990): 27–47.

Mair, Victor H. "The Mummies of East Central Asia." *Expedition* 52 (3) (2010): 23–32.

Mallory, J. P., "Bronze Age languages of the Tarim Basin." *Expedition* 52 (3), (2010): 44–53.

Mei, J. J., et al. "Archaeometallurgical Studies in China: Some Recent Developments and Challenging Issues." *Journal of Archaeological Science* 56 (2015): 221–232.

Needham, Joseph, and Kenneth Robinson. *Science and Civilisation in China, Vol. 4, Sound* (*Acoustics*). Cambridge: Cambridge University Press, 1962.

Origen. *Contra Celsum*. Translated by Philip Schaff. Buffalo: The Christian Literature Publishing Company, 1885.

Origen. *Contra Celsum*. Translated by Henry Chadwick. Cambridge: Cambridge University Press, 1965.

Panagiotidou, Olympia, and Roger Beck. *The Roman Mithras Cult: A Cognitive Approach. Scientific Studies of Religion: Inquiry and Explanation*. London; New York: Bloomsbury Academic, 2017.

Parpola, Simo. "Back to Delitzsch and Jeremias: The Relevance of the Pan Babylonian School to the MELAMMU Project." *MELAMMU Symposia IV*. Edited by A. Panaino and A. Piras. Milan: Università di Bologna & Istituto Italiano per l'Africa e l'Oriente (2004): 237–247.

Pinches, T. G. "Sabbath (Babylonian)." In *Encyclopedia of Religion and Ethics*, ed. James Hastings, 20: 889–891. Whitefish, MT: Kessinger Publishing, 2003.

Pinckney Hart, Jr., James. "The Philosophy of the Chou Yü." ProQuest Dissertations and Theses; University of Washington, 1973.

Plato. *Republic*. Translated by Benjamin Jowett. New York: The Modern Library, 1960.

Porphyry. *The Cave of the Nymphs in the Odyssey*. Buffalo: Dept. of Classics, State University of New York at Buffalo, 1969.

Rawlinson, Henry C. "On the Birs Nimrud, or the Great Temple of Borsippa." *Journal of the Royal Asiatic Society* 18 (1861): 17–18.

Rawson, J. "Carnelian Beads, Animal Figures and Exotic Vessels: Traces of Contact between the Chinese States and Inner Asia, c. 1000–650 BC." *Archaeol. China Bridg. Eurasia* 1, (2010): 1–42.

Reitzenstein, R. *Das iranische Erlösungsmysterium, Religionsgeschichtliche Untersuchungen*. Bonn: Marcus et Weber, 1921.

Rochberg-Halton, Francesca. "Benefic and Malefic Planets in Babylonian Astrology." In *Scientific Humanist: Studies in Memory of Abraham Sachs*, 319–324. Philadelphia: University of Pennsylvania, 1988.

Rubin, V. "The Concepts of Wu-Hsing and Yin-Yang." *Journal of Chinese Philosophy* (9) (1982): 131–157.

Sarianidi, V. "Togolok 21, an Indo-Iranian Temple in the Karakum." *Bulletin of the Asia Institute* 4 (1990): 159–165.

Senn, Frank C. *Christian Liturgy: Catholic and Evangelical*. Minneapolis: Fortress Press, 1997.

Sharf, R. H. "Chinese Buddhism and the Cosmology of Sympathetic Resonance." In *Coming to Terms with Chinese Buddhism*, 121–122. Honolulu: University of Hawai'i Press, 2002.

von Falkenhausen, Lothar. *Suspended Music: Chime-Bells in the Culture of Bronze Age China*. University of California Press, 1994.

Waerzeggers, Caroline, and Ronny Siebes, "An Alternative Interpretation of the Seven-Pointed Star on CBS 1766," NABU (Nouvelles Assyriologiques Brèves et Utilitaires) /2 (2007): 43–45.

Wang Yongqiang and Dang Zhihao. "A New Archaeological Discovery at Hami Wupu Eschernan Cemetery, Xinjiang." *The Western Regions Studies* (2011): 134–137.

West, Martin L. "The Babylonian Musical Notation and the Hurrian Melodic Texts." *Music and Letters* 75 (2) (1994): 161–179.

Yin Sheng-p'ing. "Investigation of the Racial Affinity of Two Western Chou Human Heads Sculpted of Shell." *Wen-wu* (Cultural Relics)(1986): 46–49.

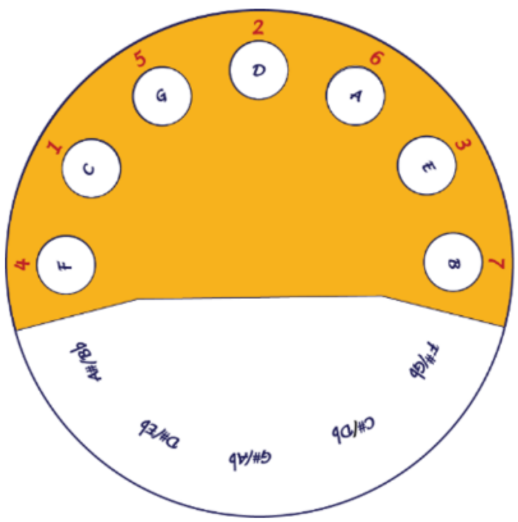
ACKNOWLEDGMENTS

The author would like to thank Victor H. Mair for sourcing and translating some of the Chinese texts cited in this paper, and for generously sharing his knowledge in related areas of study. Thanks to Buyun Chen, also, for her work in sourcing and translating some of the Chinese texts.

Many thanks also to: Richard Dumbrill, for introducing me, in 2016, to the Mesopotamian material; John Franklin of the University of Vermont, for his interest in and support of my work; Nathan Martin of the University of Michigan School of Music, for introducing me, in 2019, to the ancient Chinese *sanfen sunyi* method; Taneli Kukkonen of NYU Abu Dhabi, for helping source the translations of the Latin and Greek quotations cited in this paper; Harald Krebs of the University of Victoria School of Music, for reviewing the *Musicircle* in some of its early forms and providing useful feedback; Allan MacKinnon of Simon Fraser University, for encoraging me, over the course of many years; Susan Gerofsky of the University of British Columbia, for her support and enthusiasm. My sincere thanks also, to many of the residents of my two island communities, for their interest in and support of my work, over the course of many years. And a special thank you to my family, for believing in me.

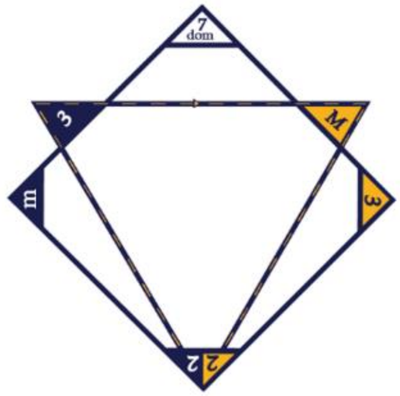
ABOUT THE AUTHOR

Sara de Rose became involved in the field of music archaeology in an unusual way. As a young adult, she taught herself music theory by identifying patterns, creating the model of the circle of fifths and the sequence 4,1,5,2,6,3,7 shown here. To use this model, she rotated the sequence (the yellow overlay) on the circle of fifths, finding the notes in any major scale.



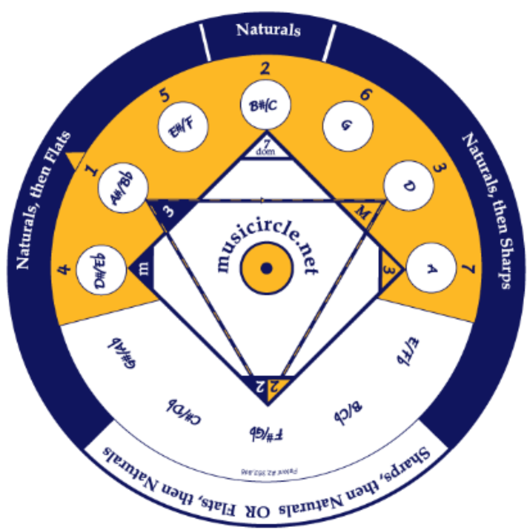
Circle of Fifths and Sequence 4,1,5,2,6,3,7

Later, de Rose added a composite shape made from a triangle and a square, shown below. The points of this shape indicate the notes in different types of chords — major, minor, complex, augmented, diminished, etc. When used with the yellow overlay, this shape allows one to see, at a glance, which chords are "in" a certain key.



Composite Shape: Triangle and Square

Later still, de Rose added an outer rim that indicates how to name of the notes in scales and chords correctly. She called her tool — which illustrates the relationship between music and geometry — the *Musicircle*.



https://musicircle.net

While creating the *Musicircle*, de Rose also developed a basic understanding of astronomy, learning how to track the planets through the constellations of the zodiac.

In 2001, she learned that the days of the week are named after the seven classical planets, but was confused by the order of the days. On a hunch, she gave each body a number indicating its apparent relative speed, coming out with the sequence 4,1,5,2,6,3,7.

Heptagram made by de Rose in 2001

This inspired de Rose to wonder about the origin of the sequence 4,1,5,2,6,3,7. Finding no written references (she was still unaware of the Mesopotamian and Chinese material, and of the writings of Origen and Cassius Dio), she decided to learn about the origin of the circle of fifths.

Using a strip of paper to represent a guitar string, she duplicated, without knowing it, the ancient Chinese up-and-down principle, coming out, once again, with the sequence 4,1,5,2,6,3,7. Then, because she was used to working with a circular model, she expressed this linear pattern in a circular format, deriving, in 2001, the heptagram shown here.

In 2002, de Rose received a patent for the *Musicircle*: https://musicircle.net. Then, 14 years later, in 2016, she became aware of tablet CBS 1766. Upon realizing that the sequence 4,1,5,2,6,3,7 — the sequence on the *Musicircle* — is written on CBS 1766, and also that the heptagram that she derived in 2001 and the heptagram drawn on CBS 1766 are virtually identical, de Rose wrote a description of how the heptagram can be derived from the simple mathematics of music (i.e., the up-and-down principle) and sent it to a music archaeologist working in the field of Mesopotamian music. Since that time, de Rose has been involved in the field of music archaeology. You can learn more about her academic work at https://musicircle.net/academic-papers-by-sara-de-rose/.

Currently, de Rose focuses on educating the public about the Mesopotamian musical system and the role of the sequence 4,1,5,2,6,3,7 in ancient philosophical tradition. She believes that incorporating this material into modern music education is essential, for three reasons:

First, the Mesopotamian musical model (formatted as the *Musicircle*) allows modern music theory to be understood as a system of simple patterns, rather than learned by rote. Second, the Mesopotamian material allows us a glimpse into deep history, explaining why the major scale has seven notes, why some notes are named as sharps and flats, and why there are twelve notes in an octave — questions that, until now, have had no satisfactory answers. Finally, the Mesopotamian material allows us to appreciate the religious/spiritual importance our distant ancestors placed on music, thereby leading us to contemplate our own unique relationship with this enduring art form.

https://musicircle.net/